QA
11
.A72

U. of M. Extension
library

# THE ARITHMETIC TEACHER

*Volume 8, number 8* DECEMBER 1961

Looking ahead at instruction in arithmetic — *John R. Clark*

Arithmetic understandings needed by elementary-school teachers — *Jack N. Sparks*

Intra-class grouping for arithmetic instruction: critique and criteria — *Harold H. Lerch*

Attitudes toward a television program—patterns in arithmetic — *George Kaprelian*

E pluribus unum—a brief discussion on the "law of one" — *John K. Reckzeh and Ernest R. Duncan*

---

*A device for teaching some number concepts,*
*Progress report on the SMSG elementary-mathematics project,*
*and the Classified Index for* 1961

AN OFFICIAL JOURNAL OF *The National Council of Teachers of Mathematics*

# THE ARITHMETIC TEACHER

DECEMBER 1961 *Volume 8, number 8*

page 385 From the editor's desk—, *E. Glenadine Gibb*
386 As we read, *E. W. Hamilton*
388 Looking ahead at instruction in arithmetic, *John R. Clark*
395 Arithmetic understandings needed by elementary-school teachers, *Jack N. Sparks*
403 Professional dates
404 Intra-class grouping for arithmetic instruction: critique and criteria, *Harold H. Lerch*
408 Attitudes toward a television program—patterns in arithmetic, *George Kaprelian*
413 E pluribus unum—a brief discussion on the "law of one," *John K. Reckzeh and Ernest R. Duncan*
416 Discovering bar graphs by degrees, *Thomas J. Jennings*
417 Books received
418 Evaluation of a mathematics program, *George F. Madaus*
422 San Diego's new arithmetic program, *J. Fred Wilson, William Self, and Don Chamberlin*
425 Don't count your chickens before they hatch, *J. William Lambert*
428 Planning an arithmetic fair, *Dorothy F. Gilman, Principal, and Members of the Faculty*
431 Just plain drill, *Josephine K. Coleman*
433 In the classroom, *edited by Edwina Deans*
The use of overlay charts, *Ruth Machlin*
436 Experimental projects and research, *edited by J. Fred Weaver*
The School Mathematics Study Group Project on Elementary-School Mathematics: A Progress Report
440 Reviews, books and materials, *edited by Clarence Ethel Hardgrove*
442 Index for 1961

THE ARITHMETIC TEACHER
*an official journal of The National Council of Teachers of Mathematics*

*Editor:*
E. GLENADINE GIBB
*State College of Iowa*
*Cedar Falls, Iowa*

*Associate Editor:*
E. W. HAMILTON
*State College of Iowa*
*Cedar Falls, Iowa*

*Assistant Editors:*
EDWINA DEANS
*U.S. Office of Education*
*Washington 25, D.C.*

CLARENCE ETHEL HARDGROVE
*Northern Illinois University*
*DeKalb, Illinois*

J. FRED WEAVER
*Boston University*
*Boston, Massachusetts*

MARGUERITE BRYDEGAARD
*San Diego State College*
*San Diego, California*

JOHN R. CLARK
*New Hope, Pennsylvania*

VINCENT J. GLENNON
*Syracuse University*
*Syracuse, New York*

All editorial correspondence, including books for review, should be addressed to the Editor. All manuscripts submitted for publication should be in duplicate copy. Advertising correspondence, subscriptions to THE ARITHMETIC TEACHER, and notice of change of address should be sent to:

THE NATIONAL COUNCIL
OF TEACHERS OF MATHEMATICS
1201 SIXTEENTH STREET, N.W.
WASHINGTON 6, D.C.

Second-class postage paid at Washington, D.C., and at additional mailing offices. Printed in U.S.A.

THE ARITHMETIC TEACHER is published monthly eight times a year, October through May. The individual subscription price of $5.00 ($1.50 to students) includes membership in the Council. (For an additional $3.00, or $1.00 to students, the subscriber may also receive THE MATHEMATICS TEACHER.)

Institutional subscription: $7.00 per year for each journal. Single copies: 85¢. Make remittance payable to The National Council of Teachers of Mathematics, 1201 Sixteenth St., N.W., Washington 6, D.C. Add 25¢ per year for mailing to Canada, 50¢ per year for mailing to other foreign countries.

OFFICERS OF THE NATIONAL COUNCIL FOR 1960–61 AND YEAR TERM EXPIRES:

*President*
Phillip S. Jones, 1962, University of Michigan, Ann Arbor, Mich.

*Past President*
Harold P. Fawcett, 1962, Ohio State University, Columbus, Ohio

*Vice-Presidents*
Clarence Ethel Hardgrove, 1962, Northern Illinois University, DeKalb, Ill.; Mildred Keiffer, 1963, Cincinnati Public Schools, Cincinnati, Ohio; Eunice Lewis, 1962, University of Oklahoma, Norman, Okla.; Bruce E. Meserve, 1963, Montclair State College, Montclair, N.J.

*Executive Secretary*
M. H. Ahrendt, 1201 Sixteenth Street, N.W., Washington 6, D.C.

*Board of Directors*
William T. Guy, Jr., 1964, Austin, Tex.; Max Beberman, 1964, Urbana, Ill.; Julius H Hlavaty, 1964, New York, N.Y.; Philip Peak, 1962, Bloomington, Ind.; Oscar F. Schaaf, 1962, Eugene, Ore.; Henry Van Engen, 1962, Madison, Wis.; J. Houston Banks, 1963, Nashville, Tenn.; Irvin H. Brune, 1963, Cedar Falls, Iowa; Robert E. K. Rourke, 1963, Kent, Conn.

Information about

**THE ARITHMETIC TEACHER**

How does an editor try to serve her public?

Read "From the Editor's Desk" on page 385.

See the information and order form at the back of this issue.

This insert is perforated for easy detachment if you wish to remove it.

# From the editor's desk—

E. GLENADINE GIBB

With this issue we are ready to bind together another volume of THE ARITHMETIC TEACHER. Throughout the year this journal has brought its readers various points of view on curriculum, teacher education, and approaches to the teaching of mathematics. It has been a reporter, reporting the results of research in elementary-school mathematics, noting the implications of these studies for making decisions about the future of mathematics in our elementary schools. It has been a teacher through its pages on which various topics in mathematics were presented. It has served as a source of information about new research, ongoing experimental programs, tested ideas to be used in the classroom, and reviews and listings of new books and other teaching materials.

Let us browse through the pages of this volume. Schaaf's "Mathematics is a cultural heritage" not only reviewed the history and development of mathematics, but also explained the inadequacy of a basic diet of "grocery store" arithmetic. We recall that the February issue brought comments from both "youngsters" and "oldsters." The younger set wanted to know more about "Unusual arithmetic." The adults were concerned with the research techniques used in the studies reported.

For many readers the March issue was a favorite. The papers on creativity and mathematics and those on the ever-persistent problems of grouping, class organization, and promotion were the features.

Although our readers have expressed their gratitude for "As we read" by E. W. Hamilton in all the issues, the questions designed to help us read the papers in the April issue with a critical eye were especially appreciated. Certainly the many aspects of problem solving have been sources of teaching difficulty for many of us.

It seemed only fitting that in May there should be articles which would help us in re-evaluating our practices in the light of new knowledge, new research, and new ways of developing old ideas. These papers along with Weaver's summary of selected research for the past year provided material for reflection and thought throughout the summer vacation.

In October "A time for decision" by Sueltz was helpful in giving direction for action resulting from our summer meditations. The November issue was a heavy one—heavy with suggestions, questions, and implications.

In 1962 THE ARITHMETIC TEACHER will strive to provide information, inspiration, and motivation for teaching elementary-school mathematics. We plan to bring you thoughtful considerations of such current issues as new instructional approaches, the changing curriculum in arithmetic, and the development of attitudes and interests. We hope to bring you reports on implementations of new programs, the evaluation of teaching innovations, and

the use of programed instruction and television. There will be summaries of research in the teaching and the learning of mathematics both on the part of children and of teachers.

We have been happy to send this issue of THE ARITHMETIC TEACHER to many of you as new readers. We hope you will decide today to become one of our regular readers in 1962.

# As we read

E. W. HAMILTON *Associate Editor*

Dr. John Clark is one of the respected elder statesmen of mathematics education. It is with considerable satisfaction that we publish, in this issue, his summary of present progress and his prediction as to the trends we will follow in the decade ahead.

This might well serve as an outline for a syllabus for teacher preparation. Certainly any course which is not implementing most of these ideas ought to be examined critically. Most of us college teachers could profit by measuring our daily, weekly, or term performance against the yardstick these comments provide.

Our students ask "When are you going to tell me what to do?" How to teach in ten easy lessons seems to be the expectation. Even our own colleagues in teacher training or in other disciplines imply something the same upon occasion.

During my first year in graduate school, I was advised against going into elementary-mathematics education by an older graduate student. That field is all worked out, there is no important research left to be done, and nobody is particularly interested in it, was the import of his argument. There was a time when you could point out in the literature, the answers students still demand. True, they were wrong answers in many respects—just as wrong then as they are now. The reason we can't tell teachers just what to do, the reason we don't all agree on what to do, is that we are alive again. More questions are being asked than can be answered in the time and with the manpower available. Arithmetic is a frontier of education again as it was in the early decades of the twentieth century, but with a different vision and a different leadership. No doubt, mistakes are being made along with much progress, but overenthusiasm is to be preferred to none at all, and the ranks of competent teachers, supervisors, and research experts are filling.

The other articles in this issue illustrate in more detail and reinforce or modify many of the points made by Dr. Clark.

Something of the size of the task ahead as we attempt to prepare for the coming program as outlined by Clark looms up as we study Sparks' article. Allowing for the argument that research instruments are far from perfect and that formal courses aren't always an adequate measure of a teacher's competence, this is still a rather sorry picture of the status of teacher preparation right up to 1959 and 1960. These measures and estimates of adequacy and of teacher preparation were made with the present curriculum and textbook material as a background, so that they reflect our shortcomings with respect to Clark's first point, that of greatly improved scholarship. His fifth point, increased use of certain new mathematical topics, widens further the gap between goal and actual-

ity. Sparks' references, as well as his text, should be of interest to students, those interested in research, professional preparation, and revision of college curriculum.

One phase of the fourth point in Clark's list is elaborated further by Lerch. His description of the purposes of subgrouping to permit easier communication, to reduce interaction, and to increase pupil success sounds so good that we have to read on to see why it has any opponents. We begin to suspect that the objections stem from poor grouping practice and lack of flexibility in administration. Considering how hard it is to break the lockstep enough to get and keep intact reasonably well-defined groups of children for research studies, it is not surprising that all sorts of contingencies take precedence with the result the subgroups are no more homogeneous with respect to traits in arithmetic than their parent group. Subgrouping conducted thus multiplies the work without any compensating gain. With good motivation this ought to be a fertile field for programed learning and teaching machines. Many of the objections to subgrouping are removed almost at once, and many of the advantages should be accessible by this means.

This also touches upon Clark's point number six, but another aspect of the "newer tools of learning" is elaborated by Kaprelian. It is easy to fail to distinguish, as well as the topic deserves, between the children's reactions to television as a medium and their reactions to curricular materials considerably different than the traditional. Read carefully. There is interesting information on both subjects. The 5 to 10 per cent of dissenters compares quite closely with unfavorables on questionnaires I have used with college students. Apparently it is not only the poor that we have always with us!

The article by Jennings pursues the much-discussed subject of discovery. Discovery seems to defy definition and to have no limits. It is already old but always new, simply because it is the very essence of learning. Watch either a research scientist or his two-year-old grandchild. All our techniques, materials, and organizations are merely means to that end. Let us continually examine them to see that they do not obscure that end or usurp its place! Clark's point about further discovery being impossible without concepts is justification for concern that teachers understand, use, and guide toward such notions as Reckzeh and Duncan develop. The law of one or the multiplicative identity is one of those productive concepts which contribute to understanding and further discovery at many levels.

Lambert's description of children's interest in hatching chickens and all the practical problems that had to be met illustrates at a very elementary level the formation of concepts by experience and observation. The motivational effects of two widely different activities are reported by Gilman and by Coleman. Something to work for in the way of recognition and tangible record is the essence of both. This sort of thing has been advocated in the pedagogical literature for years. How nice it will be when a whole generation of elementary teachers arises who knew nothing but good teaching when they themselves were children! They will look upon such articles as these as too obvious—but, until that day arrives, we hope more people will try out such things and discover their worth.

The articles by Madaus and by Wilson round out the issue. Madaus lists many of the things we should keep in mind with regard to continuous informal evaluation as well as occasional formal testing. Wilson describes the organization adopted by a large system to meet the challenge of new material and new ideas. That task will be faced by many school systems in the years just ahead—this description can serve as a model or a point of departure. These are problems whether we know the way the wind is blowing or not—or even if we as individuals happen to be caught in a dead calm.

# Looking ahead at instruction in arithmetic*

JOHN R. CLARK
*Consultant in Mathematics Education, New Hope, Pennsylvania*

Herbert Hannon of Western Michigan University tells a story of a real-estate agent who was scrupulously honest with his clients. Upon showing a client a piece of property, he pointed out that the house was satisfactory, but that the location had a serious drawback. Two blocks to the west of the house was a brewery; two blocks to the east was a slaughter house. The client agreed that the location had disadvantages, but asked if there was no advantage to it.

"Oh, yes," replied the agent. "The advantage is that you can tell which way the wind is blowing."

Can we tell which way the wind is blowing in the teaching of arithmetic? The writer believes that, as a result of a decade of experimentation, involving teachers, psychologists, and mathematicians, it is now possible to make some quite dependable "predictions of things to come" in the teaching of arithmetic—to chart, as it were, the arithmetic program of the coming decade.

Before discussing major aspects of a modern program in arithmetic, let us acknowledge our debt and gratitude to the many groups of researchers (such as those sponsored by Davis, Glennon, Page, Moser, Begle, Weaver, Gundlach, and others) who have investigated the problem of ability to learn subject matter above and beyond that found in standard arithmetic curricula. These groups have clearly demonstrated that pupils (selected groups of pupils) can learn, and enjoy learning, subject matter traditionally considered too difficult for pupils of the grades in which the experimental materials have been used. These leaders agree that determination of what pupils of a given grade *can* learn under the guidance of superior teachers (superior in scholarship and in the art of teaching) is only the first step in determining what pupils *should* learn. The fact that a mathematical game has proved exciting to pupils of a given grade is, of itself, no evidence that a mathematical game should replace subject matter currently considered best for that grade.

Selection or determination of what subject matter *should* be taught requires the combined judgment of teachers, psychologists, mathematicians, and educational philosophers. Society, in the last resort, must make this decision. Since we live in a changing culture, the arithmetic curriculum can remain static for only short periods of time—perhaps for a decade.

Six of the major aspects of the coming program, as we see them are:

1 Greatly improved scholarship of teachers in the nature or structure of arithmetic;
2 Deeper conviction in the public mind of the increasing significance of arithmetic in our culture;
3 Greater emphasis upon thinking and discovery in arithmetic learning;
4 More consideration of (and provision for) the implications of the wide range of ability in any age group;

* A paper read at the May, 1961, Syracuse meeting, Association of Mathematics Teachers of New York State.

5 Increased use of certain mathematical topics not at present widely included in the arithmetic curriculum;
6 Wider use of newer tools of learning.

Now to a consideration of the six major aspects of the instructional program in arithmetic in the coming decade.

1 *Improved scholarship in arithmetic.* Teachers will surely have a better understanding of

*a*) number and numeral systems;
*b*) definitions of operations and the laws governing them;
*c*) fractions as numbers and numerals;
*d*) estimation (with rounded numbers) and mental computation;
*e*) the special roles of the numbers zero and one;
*f*) rates, ratios, and proportions;
*g*) the solution of equations, especially by using the concepts of inverse operation;
*h*) statistics (collection, tabulation, graphic representation of data, range, central tendency, variability, and correlation);
*i*) informal and deductive geometry;
*j*) applications of arithmetic to science, business, and industry.

Through subject-matter courses in arithmetic, specifically designed for teachers (both in-training and in-service), taught by instructors with a high degree of scholarship in mathematics and with intimate familiarity with the problems of the elementary school, this essential preparation for teaching arithmetic can and will be achieved. In the coming decade teachers, unlike teachers of the past decade, will be "at home" in arithmetic. Though not mathematicians, they will be "mathematically minded."

2 *Deeper realization among lay leaders of the increasingly significant role of arithmetic in our culture.* These leaders are now demanding, and will continue to demand, a higher level of achievement in arithmetic. They are convinced that we teachers have been willing to accept an inferior quality of learning. Believing that our national security is dependent (among other factors) upon the quality of instruction in mathematics and science, these leaders will insist that each elementary school have the service of one or more teachers with special competence in each of these areas. These specialists in arithmetic, just as the specialists in art and music, will work both with pupils and teachers.

3 *Greater emphasis upon learning by* thinking *and* discovering. One *thinks* when one wishes to respond to a problem-situation for which he has no memorized response. Thinking deals with ideas or concepts and the relationships among them. To illustrate, consider the problem of finding the missing numbers in the sequence 2, 3, 5, 7, 11, . . . . The pupil who has the concept of prime numbers, can interpret the sequence as a sequence of primes, and then has little difficulty in discovering subsequent numbers: 13, 17, 19, 23, etc. Without the concept of primes, no correct response is possible, even by a very bright learner. Hence an acceptable response to the problem-situation is dependent upon the learner having the concept of prime numbers.

To respond by immediate recall to "What is the sum of 128 and 23?" would be impossible for most of us, even though we had previously made the response. We did not drill on it, did not *memorize* it. The number of such responses which we may need to make is so very, very great that it would be futile to try to memorize them. Having the concept of addition as "counting on," we easily *think out* the sum, in some such way as:

> 128 and 20 is 148; 148 and 3 is 151, or 120 and 20 is 140; 8 and 3 is 11; 140 and 11 is 151

(or by any other application of the concept of association in addition).

The learner who conceives of subtraction as the inverse of addition is easily able to *think out* the remainder when 23 is

subtracted from 151. He "counts back" or "counts off." He may think:

151 − 21 is 130; 130 − 2 is 128; or
151 − 20 is 131; 131 − 3 is 128

(or by any other application of the concept of association in subtraction).

The valid ways of thinking in performing an operation make it easy for the learner to "bring forth" multitudes of responses which he has never previously made.

Guided by a teacher who understands arithmetic and who conceives of learning as thinking, classroom learning of arithmetic becomes lively, challenging, and rewarding. Motivation is generated; surprisingly clever and ingenious solutions are proposed. Only in such a "climate of learning" can learning be truly creative.

Creative teaching is the very antithesis of the "show and tell," the "this is the way to do it," the "get busy and learn this" type of teaching. It is a search to discover answers, engaged in by both teacher and pupils. Often questions are proposed which the teacher cannot answer. Instead of being shocked or insecure, she says the equivalent of "That's a fine question! Frankly, I don't know the answer. Let's try it together. Who has an idea that might help us to find the answer?" Apropos of this situation, Franz Hohn, in the March, 1961, issue of THE ARITHMETIC TEACHER, says that some of the finest creative teaching comes out of such mutual searching for solutions to problems the pupils have raised. Hohn states:

> Experience in discovery in a wide range of situations is the critical factor in the development of genuine mathematical thinking.

The summary statement in the TWENTY-FIRST YEARBOOK of the National Council of Teachers of Mathematics predicts that "instruction in mathematics for general education will come to rely more and more on the implications of the following:

- learning is thinking
- successful thinking is possible at any grade or achievement level
- successful thinking is heavily dependent on concepts and relationships
- the satisfactions following successful thinking provide enduring enrichment for the learner."

4 *Implications of the range ability in heterogeneous groups of a given age.* It is futile to attempt to teach a heterogeneous group of pupils of any grade the same subject matter, at the same time, and expect an equal degree of achievement. If the level of difficulty of subject matter is such as to challenge the faster learners, the slower learners are frustrated. If the level of difficulty is appropriate for the slower learners, the faster learners will not only be unchallenged—they will be bored and disinterested.

Curriculum makers and textbook authors are productively engaged in designing instructional materials of optimum difficulty to challenge both groups. Optimum difficulty is conceived to mean that the goals set are attainable only by effort. Whether the pupil is a slow learner or a fast learner, he should be expected to succeed at each task most of the time.

Many schools, having two or more sections of pupils in a given grade, classify them into slower and faster learners. At this point, teaching-practice varies. To provide for faster learners, some schools use subject matter of greater depth or difficulty, allowing for no acceleration gradewise. Others increase the pace, completing, say, three years of standard matter in two years. To provide for slower learners, practically all schools plan a minimum curriculum of less than average difficulty.

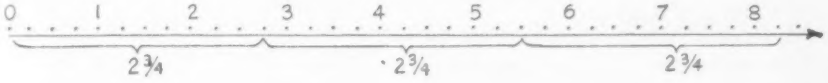
Other schools are experimenting with the concept of team learning, in which two (or more) pupils at approximately the same level of aptitude and achievement are organized into a working team. In a class of thirty pupils there may be eight or ten teams, all working on different jobs (assignments) and progressing at different rates. To date, no one solution has general acceptance, but the over-all agreement is that faster learners as well as slower

learners need special teacher attention.

In the coming decade we shall see much research on this problem. My prediction is that the profession will come to favor homogeneous grouping or sectioning, use subject matter of two or more levels of difficulty, and work hard at producing challenging material of instruction for both slower and faster learners. Our objective will be to challenge all pupils so that "each may succeed according to his talents."

5 *Topics due for wider use in the arithmetic curriculum.* We may expect wider use of

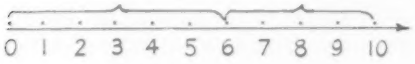
*a*) number lines (geometric scales);
*b*) frame arithmetic;
*c*) number sentences;
*d*) estimation;




*e*) mental computation;
*f*) equivalent and nonequivalent number expressions.

A brief comment about each follows.

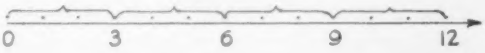
*a*) *Number lines.* Number lines help the learner to visualize both number facts and the concept of operations. Equally spaced points on a line, beginning at zero, are represented by numerals, as shown here.




To add 6 and 4, begin at 6 and count on 4 spaces, arriving at 10 showing that $6+4=10$, and that addition is a joining or putting together operation, a counting on from one addend through another. The commutative property of addition may also be "seen" by beginning at 4 and counting on 6 spaces, arriving at 10.

The number line helps the learner to conceive of subtraction as the inverse of addition. To subtract 4 from 10, begin at 10, and count back or count off 4, showing that $10-4=6$. Again the number line shows a number fact and reinforces a concept.

The number line helps to visualize a multiplication fact and the concept of multiplication.




Thus $4\times3$ is seen to be 12; the repeated addend is emphasized. Thus, multiplication is associated with combining a known number of groups of a given size. It isn't necessary to point out that the size of the group may be represented by a common fraction, decimal, or mixed number. Here the multiplicand $2\frac{3}{4}$ is repeated 3 times, showing that $3\times2\frac{3}{4}=8\frac{1}{4}$.

Since division should come to mean the inverse of multiplication, this same number line shows that $2\frac{3}{4}$ is contained in $8\frac{1}{4}$ three times. We begin at the dividend $8\frac{1}{4}$ and mark off as many $2\frac{3}{4}$'s as possible. Since the marking off or measuring stops at zero, there is a remainder of zero. Again the use of the number line in a measurement division shows both a number fact and reinforces the measurement concept of division. (We know of no way to use a number line in a partition division.)

*b*) *Number sentences.* In former years, pupils were given much practice in *problem analysis*—in stating the relationship among the concepts in a problem. To illustrate, in the problem "What is the cost of 3 loaves of bread at 24 cents a loaf?" there are three concepts: (1) cost of all, (2) cost of one, and (3) number of loaves bought. These concepts are related: cost equals number of loaves bought times the cost of one loaf. We have analyzed the problem, and stated the relationship among the concepts, in a "word" sentence. (Some of us were fortunate enough to

have had teachers who required us to make such problem analyses.)

Instead of expressing the relationship among the concepts of a problem in a complete "word" sentence, teachers now prefer that pupils use a "number" sentence, such as

$$N=3\times24,$$

in which $N$ symbolizes the cost, 3 the number bought, and 24 the cost of one loaf.

For the problem "Ella has 37 cents. She wants to buy a book which costs 50 cents. How much more money does she need?" the word statement of the relationship among the three concepts is "37 cents plus the amount she needs equals 50 cents." The briefer statement or number sentence way of stating the relationship is

$$37+N=50.$$

In number sentences, number symbols and signs of operations replace the words used in the word sentences. Both stress analysis and statement of relationship among the concepts in a problem.

Formerly teachers were reluctant to use letters as number symbols. Now teachers find their use productive.

To go back to the number sentence $37+N=50$, the pupil is taught that $37+N=50$ would be a false statement if $N$ is replaced by any number other than 13. The number sentence is true if and only if $N$ is replaced by 13.

Why call $37+N=50$ a number sentence rather than an equation? We do it partly because of its genesis, an abbreviation of a word sentence used in problem analysis. But there is another reason. Teachers wish pupils to find the number that $N$ stands for, not by using the subtraction axiom of algebra, but rather by using the principle "an unknown addend ($N$) equals the known sum of two addends (50) minus the known addend (37)."

*c) Frame arithmetic.* Number sentences, such as $\square+17=30$ and $6\times\triangle=50$, are often called *frame* arithmetic. In "$\square+17=30$" the pupil is asked, "If you put 10 in the square, would the statement be true or false?" By questioning, the pupil concludes that the statement would be true if and only if 13 is put in the square.

Exercises entitled, "Tell some pairs of numbers you could put in these frames to make true statements" are interesting and challenging.

$$\square+\triangle=20 \qquad \square-\triangle=9$$
$$\square\times\triangle=10 \qquad \square\div\triangle=4$$

The making of tables, such as this one for $\square+\triangle=10$,

| $\square$ | $\triangle$ |
|---|---|
| 0 | 10 |
| 1 | 9 |
| 2 | 8 |
| 3 | 7 |

serves to emphasize that there are many solutions to some problems.

In cases, such as

$$\square+\triangle=\triangle+\square \qquad \square+\square+\square=3\times\square,$$

the number of solutions is limitless.

Frame arithmetic doubtless will be used extensively in teaching basic facts in each of the four operations.

*d) Mental computation.* Teachers are increasingly aware of the importance of mental computation. Most of the computations of everyday experience, for which no permanent record is needed, can and should be done by mental computation. A survey of the arithmetic behavior of adults who were considered school failures in written computation shows that they have taught themselves to compute mentally. We have seen such illustrations as:

- $\frac{1}{2}$ of 156 is $\frac{1}{2}$ of 100 (50), and $\frac{1}{2}$ of 50 (25), and $\frac{1}{2}$ of 6 (3), or 78.
- 37 from 61 is derived by thinking, 31 from 61 is 30; 6 from 30 is 24.
- $5\times24$ is $5\times20$ (100) and $5\times4$ (20), so $5\times24$ is $100+20$ or 120.

- 97+28 is 97+20 (117), and 8 more are 125.
- $6\frac{1}{4}-2\frac{1}{2}$ is $6\frac{1}{4}-2$ is $4\frac{1}{4}$; $4\frac{1}{4}-\frac{1}{2}$ is $3\frac{3}{4}$.
- $3\frac{1}{2}\times 80$ cents is $3\times 80$ cents ($2.40), and 40 cents more is $2.80.

The laws of operation (used intuitively by these persons) provide many correct ways of thinking about sums, differences, products, and quotients.

A point that I should like to stress is that mental computation is not to be confused with visualizing the digits in an algorism. To subtract 473 from 1000 by mental computation, one does not image

$$\begin{array}{r} 1000 \\ 473 \\ \hline 527 \end{array}.$$

Rather he thinks in some such way as:

400 from 1000 is 600; 70 from 600 is 530; 3 from 530 is 527. Mental computation is both easier and more accurate than computation requiring imagery of the digits as they appear in written computation.

In the next decade, mental computation will surely be stressed.

*e) Estimation of sums, differences, products, and quotients.* To estimate the total cost of four articles which cost $1.80, $3.20, $1.98, and $4.10 respectively, one thinks $2+$3+$2+$4, or $11.

To estimate the change after one spends $3.95 and gives the clerk a ten-dollar bill, one thinks "10 dollars−4 dollars is 6 dollars," etc. Thus estimation requires the use of round numbers.

The frequency of the need to estimate is indeed great. Instruction should certainly facilitate competence in estimating. Some illustrations follow:

- The product of $7\frac{7}{8}$ and $2\frac{1}{4}$ is about $8\times 2$, or 16.
- The quotient of 20.2 divided by 5.4 is about $20\div 5$, or 4.
- The number of dozen in 135 is more than 10, because $10\times 12$ is 120. It is more than 11.
- The product of 7.84 and 2.96 is close to $8\times 3$, or 24.

Estimation sharpens the learner's number sense, or judgment, in arriving at reasonable answers, and may be used as a check on the correctness of written computation.

Growth in ability to estimate, with smaller and smaller errors, is coming to be accepted as a significant objective of instruction in arithmetic.

*f) Number expressions: equivalent and nonequivalent.* Numbers are abstract; they are ideas, existing only in the mind; they cannot be seen or written. In order to make records about numbers, we use symbols. Symbolic representation of the number of things in a dozen includes, among many others, these number expressions:

12 $10+2$ $3\times 4$ $11+1$ $\frac{1}{2}$ of 24 $20-8$, etc.

The "12" is the simplest number expression (or symbolization) of the number of things in a dozen; all of them are equivalent. A number expression indicating that an operation is to be performed is *simplified* when the indicated operations have been performed.

Most teachers prefer calling 12, $10+2$, $3\times 4$, $11+1$, etc. equivalent number expressions to calling them *different names* for the same number.

We see that $12\div\frac{2}{3}$, $12\times\frac{3}{2}$, $\frac{1}{2}$ of $(12\times 3)$, $\frac{3}{2}(12)$, and $\frac{12}{2/3}$ are equivalent number expressions; all are equivalent to 18.

We symbolize the cost of 6 tablets at 12 cents each as "$6\times 12$ cents," but we *prefer* the expression "72 cents." What we call computation is really the process of replacing one number expression by an equivalent one in simpler (or simplest) form.

The average (mean) of 6, 7, 9, 14, and 18 is symbolized by the number expression

$$\frac{\text{“}6+7+9+14+18\text{ ”}}{5}.$$

However, we prefer the expression "$\frac{54}{5}$";

still more preferable is the expression "$10\frac{4}{5}$." Technically speaking, then, computation is concerned with replacing a given number expression by an equivalent one, simpler in form and more easily comprehended.

Practice in determining which of two (or more) number expressions represents the larger (or largest) number, is highly productive and interesting to the learner.

The number expression $5\times12$ symbolizes a number smaller than $6\times11$. Why? By performing the indicated operations we see that 60 is less than 66; and we symbolize the relation by $(5\times12)<(6\times11)$.

Other topics appropriate for faster learners include prime numbers, least common multiple, least common denominator, largest common factor, integers, rational and irrational numbers, and numeral systems with bases other than ten.

6 *Wider use of newer tools of learning.* Continued experimentation with and evaluation of new tools of learning are predicted. These tools include: programed learning, televised instruction, filmstrips, and auditory records.

These new tools have enthusiastic advocates. Some of the tools are designed to facilitate individual progress; some are for group teaching. We comment below upon two of them.

Programed learning, through a specially designed textbook (with or without a machine) is designed to present the subject matter in many small sequentially arranged steps. Pupil responses require little writing and are so presented that the learner can check immediately the correctness of his response. Programed materials need to be prepared jointly by specialists in mathematics education and by technicians in programing.

When used for establishing computational skills, well-designed programed learning will make it possible for a pupil to practice those, and only those, skills in which he lacks mastery, and to proceed at his own rate.

My experience with televised instruction (largely at Hagerstown) suggests that television is a potent tool for upgrading instruction. When planned by committees representing each of the grades, and taught by superior teachers, both motivation and achievement are increased.

In the next decade these new tools will increasingly supplement, and, in some cases, replace the learning aids with which teachers are currently familiar.

In summary, arithmetic programs in the next decade will (1) be directed by teachers of high scholarship; (2) be better supported by the lay public; (3) give greater emphasis to learning by thinking; (4) make better provisions for the range of ability in heterogeneous groups of pupils; (5) incorporate topics not at present generally found in school curricula; and (6) include experiments with and evaluation of newer tools of learning. Programs will be upgraded in content and in method, approximating the programs currently considered superior. It will be a challenging decade for us teachers who believe that arithmetic learning can be creative and highly rewarding to our pupils. We shall build a more mathematically-minded young America.

---

"The fear of mathematics is a tradition handed down from the days when the majority of teachers knew little about human nature, and nothing at all about the nature of mathematics itself."—W. W. SAWYER in *Mathematician's Delight*

# Arithmetic understandings needed by elementary-school teachers

JACK N. SPARKS
*Colorado State College, Greeley, Colorado*
*Professor Sparks is a member of the Division of Education, Colorado State College.*

The problem of an adequate background in mathematics for elementary teachers has received considerable attention in recent years with apparently little resulting change in the training actually received. Those investigators who have attacked the problem have variously considered four aspects:

1 What mathematical understandings should elementary teachers have?
2 What mathematical understandings do teachers have?
3 What training in mathematics are elementary teachers getting?
4 How can the mathematical understandings of elementary teachers be improved?

This author contends, however, that research to date has been too superficial and limited in scope to offer substantial assistance in planning programs of teacher education. To demonstrate what is meant, let us examine the state of knowledge with respect to each of the four aspects of the problem.

### What mathematical understandings should elementary teachers have?

This would seem to be the starting point for any plan to improve arithmetic teaching. Even here, however, about the only complete agreement to be found is that teachers need to know more than they do. The concern about the mathematical training of elementary-school teachers is indicated by the large amount of space devoted to discussions of the topic in the journals in recent years. Stone,[1] in presenting fundamental issues in the teaching of elementary-school mathematics, included the problem of recruiting and training better teachers as one of the five. He said that:

> The new mathematical training which may be developed for future grade-school teachers should give them an appreciation and understanding of the subjects they may be expected to teach and as a result should inspire them with a certain degree of respect and admiration for mathematics, if not a real liking for it.

It is not difficult to find many statements such as Stone's in the literature. It is more difficult to find specific statements detailing the arithmetical concepts teachers should have, and it is almost impossible to find any research to support a statement about needed concepts. A few polls of "experts" will be reported in a later section.

Among the more definite statements of what teachers should know are those made by F. J. Mueller in a pair of recent articles. Mueller wrote that teachers need:

1 A thorough knowledge of our system of number notation;
2 A knowledge of algorisms;
3 A knowledge of the rules of computation;

[1] M. H. Stone, "Fundamental Issues in the Teaching of Elementary School Mathematics," THE ARITHMETIC TEACHER, VI (October, 1959), 177–9.

4 A knowledge of the relations of whole numbers, fractions, mixed numbers, and per cents.[2]

In another article[3] he expresses his belief that elementary teachers need to be able to find answers that they can use to meet these everyday questions:

"Why do we carry?"
"How can we subtract a 6 in the ones' place from a 3?"
"Why do we invert the divisor?"
"Why does the decimal point in the answer go there?"

In the last section of this paper recommendations for mathematics courses for teachers of arithmetic will be reviewed. These recommendations present a more detailed picture of what the experts think arithmetic teachers should know. Nevertheless, it should be noted at this point that a well-planned attack on this aspect would of necessity include:

1 A comprehensive review of competencies desired for youngsters;
2 A compilation of the competencies the above would seem to imply for teachers;
3 A program of teacher education based upon 2;
4 An experimental study designed to test the efficiency of the program outlined.

It would seem that at present we have learned how to do only 1, 2, and 3.

### What mathematical understandings do elementary teachers have?

A number of studies of mathematical understandings possessed by arithmetic teachers have been reported in the past 15 years, most of them tending more to reveal the mathematical concepts arithmetic teachers do *not* possess. The first of these was done by Glennon in 1948. Glennon[4] constructed a test of basic mathematical understandings and administered it to 144 freshmen and 172 seniors in teachers colleges. The freshmen averaged about 44 per cent correct and the seniors averaged 43 per cent correct. He also found that the particular arithmetic courses taken by teachers in-service did not cause them to score better than other teachers in-service who had not taken the same courses.

A number of studies bearing on this problem were reported in 1953. Phillips[5] reported an investigation of the arithmetical ability of students entering the course called "Arithmetic for Teachers" at the University of Illinois. The median score of these students on the *Hundred-Problem Arithmetic Test*[6] was 75 per cent. According to the manual for this test, the median for the eighth grade is 62 per cent. Phillips concluded:

1 The amount of elementary- and high-school mathematics completed gives little indication of the achievement in meaning, understanding, and mechanical mastery. Almost all had had algebra and plane geometry.
2 Lack of achievement in mechanical mastery begins with the topic of fractions and continues with decimal fractions and per cents.
3 Problem solving achievement involving measurement, fractions, and per cent is very low.
4 There is a higher competence in algebra than in geometry.
5 Achievement in the meaning and understanding of arithmetic is very low.

Orleans[7] developed a free-answer-type test designed to cover understanding of arithmetic concepts. The test was administered in the spring of 1950 to 722 subjects, some of whom were preparing to

[2] Francis J. Mueller, "Arithmetic Preparation for Elementary Teachers," *Education*, LXXIX (January, 1959), 299–302.

[3] Francis J. Mueller, "Arithmetic and Teacher Preparation," The Mathematics Teacher, LII (November, 1959), 572–3.

[4] Vincent J. Glennon, "A Study in Needed Redirection in the Preparation of Teachers of Arithmetic," The Mathematics Teacher, XLII (December, 1949), 389–96.

[5] Clarence Phillips, "Background and Mathematical Achievement of Elementary Education Students in Arithmetic for Teachers," *School Science and Mathematics*, LIII (January, 1953), 48–52.

[6] Published by R. Schorling and J. Clark, World Book Co., 1938.

[7] Orleans and Wandt, "The Understandings of Arithmetic Possessed by Teachers," *Elementary School Journal*, LIII (May, 1953), 501–7.

teach and some of whom were teaching. A multiple-choice test was prepared with wrong choices taken from the most frequently occurring wrong responses obtained from the free-answer test. The resulting instrument was administered to 322 teachers attending summer school at the University of Colorado, the University of Illinois, and the University of Houston. None of the groups had a mean score as high as 60 per cent right on the test. Orleans proposed that such a state of knowledge on the part of teachers might result from the practice of teaching a short-cut process as though it were the process itself.

In a summary of research also published in 1953, Morton[8] reported that he had administered a popular standardized arithmetic test to 405 elementary-school teachers and found 55, or 13.6 per cent of them, to be below the eighth-grade norm.

Weaver[9] administered the Glennon test to four groups of upperclassmen prior to taking a course entitled "Methods of Teaching Arithmetic." The average score of 44.6 on the test as a whole represented only slightly more than 55 per cent of the total items.

Mueller and Moser,[10] generalizing information from several sources, listed the following deficiencies in about one-half of the students at State Teachers College, Towson, Maryland:

1 The number system is seldom thought of as related to the development of culture.
2 Except for the obvious rhythmical pattern suggested by the number words, the freshmen do not see structure in the number system.
3 The approach to quantitative computation is almost completely stereotyped.
4 The students are exceedingly dependent upon the written algorisms.

More recent research does not indicate any marked improvement in the arithmetical competencies of elementary-school teachers. O'Donnell[11] gave the *California Achievement Test*, Mathematics Section, Grades 9–14, Form W, to 109 college seniors in elementary education in an attempt to determine the level of proficiency. Remmers' "Scale to Measure Attitude Toward Any School Subject" was also administered, and each subject was interviewed personally. At the time of the interview, which was taped, subjects were asked to rework ten of the problems. The interviewer used a checklist to evaluate the problem-solving behavior he observed. The results were as follows:

1 The mean score was beyond the twelfth-grade level.
2 Students appeared to recognize their own strengths and weaknesses.
3 There was no evidence that taking the course "Teaching of Arithmetic" appeared to change arithmetic achievement or problem-solving ability.
4 The mean score was higher than the norm on the Remmers scale.
5 Students were more proficient when arithmetical computations appeared in isolated forms. The problems became more difficult when they required reading and interpretation.

Bean[12] used the Glennon test in a study sponsored by the Utah Educational Research Council. A questionnaire was devised to determine the relationship between test scores and various criteria deemed relevant to the problem:

1 Teachers' self-perceptions of their mathematical competence both before and after the test;

[8] R. L. Morton, *Teaching Arithmetic, What Research Says to the Teacher*, No. 2, Department of Classroom Teachers and AERA of NEA, p. 28.

[9] J. Fred Weaver, "A Crucial Problem in the Preparation of Elementary School Teachers," *Elementary School Journal*, LVI (February, 1956), 255–61.

[10] Francis J. Mueller and Harold Moser, "Background Mathematics for Teachers of Arithmetic," *Emerging Practices in Mathematics Education*, TWENTY-SECOND YEARBOOK of the National Council of Teachers of Mathematics (Washington, D.C.: the National Council, 1957), pp. 183–4.

[11] J. R. O'Donnell, "Levels of Arithmetical Achievement, Attitudes Toward Arithmetic, and Problem Solving Behavior Shown by Prospective Elementary School Teachers," Unpublished Ed. D. Thesis, Penn State, 1958.

[12] John E. Bean, "Arithmetical Understandings of Elementary School Teachers," *Elementary School Journal*, LIX (May, 1959), 447–50.

2 Teachers' attitudes toward the teaching of arithmetic.

The study was conducted in seven school districts with 450 teachers. A mean score of 52.46 or 65.58 per cent was obtained. Other results were:

1 As teachers gain experience (up to 11–15 years), there is a small cumulative increase in arithmetical understanding.
2 Teachers with graduate work scored higher.
3 Teachers in the intermediate grades scored higher on the decimal system of notation and operations in integers. Bean considered the results to indicate that:
   *a*) Teachers were low in ability to handle decimal processes.
   *b*) Most teachers have an adequate understanding of place-value relationships in the decimal system of notation.
   *c*) Teachers need a more thorough understanding of quantitative relationships among fractions.
   *d*) Teachers were able to explain the underlying principles (of computations) for only 60 per cent of the items.

The most general conclusions that can be drawn from these studies are that teachers are considered by all of these authors to be:

1 Particularly deficient in understanding of underlying principles;
2 Deficient in computational skills, particularly with decimals and per cents;
3 Deficient in problem-solving ability.

### What training in mathematics are elementary teachers getting?

In general, the mathematics, if any, taught to prospective elementary-school teachers has been the traditional college algebra, trigonometry, and analytic geometry series. The results of a survey of 129 state teachers colleges reported by Grossnickle[13] in the 50th Yearbook of the NSSE illustrate the situation:

1 A course in general mathematics is the predominant subject matter in required background mathematics.
2 At least one-third of these colleges do not require a course in the teaching of arithmetic.
3 The average total requirement in mathematics to graduate is about three semester-hours.

Layton[14] surveyed the situation in 85 teacher-training colleges in 45 states. The mean number of semester-hours in mathematics for prospective elementary-school teachers was 1.63. The mean number of semester-hours in arithmetic methods work in these institutions was 0.47.

The chapter entitled "Background Mathematics for Elementary Teachers" by Ruddell, Dutton, and Reckzeh in the TWENTY-FIFTH YEARBOOK of the National Council of Teachers of Mathematics[15] includes a report of a study of the mathematics requirements at 96 teacher-training institutions. The writers examined the 1957–58 catalogues of selected institutions. Sixty per cent of the institutions were found to require for graduation a three-semester-hour course in general mathematics, and 56 per cent required a course in teaching arithmetic. Only 22 per cent of the colleges required both a background course in mathematics and an arithmetic methods course.

A number of reports of programs thought to be more suitable can be found in the literature. In the 50th Yearbook of the NSSE, Mueller and Moser[16] described a course taught at the State Teachers

[13] F. E. Grossnickle, "The Training of Teachers in Arithmetic," *The Teaching of Arithmetic*, 50th Yearbook of NSSE, Part II.

[14] Willima Layton, "Mathematical Training Prescribed by Teachers Colleges in the Preparation of Elementary Teachers," THE MATHEMATICS TEACHER, XLIV (December, 1951), 551–6.

[15] A. K. Ruddell, W. Dutton, and J. Reckzeh, "Background Mathematics for Elementary Teachers," in *Instruction in Arithmetic*, TWENTY-FIFTH YEARBOOK of the National Council of Teachers of Mathematics (Washington, D.C.: the National Council, 1960), p. 301–306.

[16] Mueller and Moser, *op. cit.*, p. 184.

College, Towson, Maryland, which had roughly the following structure:

1 The historical development of systems of number;
2 The principle of structure in number systems;
3 The fundamental processes;
4 Fractions.

The content course offered at Millersville State Teachers College was described by Boyer in the TWENTY-SECOND YEARBOOK of the NCTM.[17] Intermediate teachers at that institution get one three-hour course in content mathematics and one three-hour course devoted one-half time to each content and methods. The course is divided into four parts: arithmetic, algebra, geometry, and trigonometry.

In 1958, Waggoner[18] reported an effort at Central Michigan College to improve the arithmetical preparation for prospective elementary-school teachers. The prospective teachers must complete a course called "Remedial and Functional Mathematics." This course is designed to alleviate any weaknesses in the fundamental processes of arithmetic, to provide the necessary mathematical tools for the work of the elementary curriculum, and to acquaint the student with some of the social and economic applications of mathematics. The functional section of the course consists of six units: (1) the history of mathematics and our number system, (2) roots and powers, (3) measurement, (4) graphs, (5) statistics, and (6) applications. In an attempt to evaluate the effectiveness of the program, the *Davis Test of Functional Competence in Mathematics* was administered at the beginning and end of the course as taught by four different instructors of five sections. The mean score increased from the 50th percentile (on end-of-the-year norms for Grade 12) to the 71st percentile.

The evidence available indicates that although most elementary-school teachers get very little or very inappropriate training in mathematics, there is an increasing number of attempts to provide more valuable content and methods courses. As a matter of fact, private conversations indicate that most institutions are searching for an appropriate program.

### How can mathematical understandings of elementary-school teachers be improved?

As good a place as any to begin the discussion of this topic is with the general recommendations of Grossnickle and the specific recommendations of Newsom in the 50th Yearbook of the NSSE. Grossnickle[19] advocated that:

1 Teachers should not be certified to teach arithmetic in the elementary school without one course in methods.
2 If the prospective teacher has had no mathematics since eighth grade, he should have a background course.
3 The background provided should be different for teachers of the three groups: K–3, 4–6, and 7–8.
4 Elementary-school teachers really need six hours in background mathematics.
5 The course on the teaching of arithmetic should treat both social applications and mathematical meanings at each grade level.

Newsom's[20] discussion of the mathematical background needed by teachers of arithmetic included a number of factors to be considered in the process of setting up a course in background mathematics:

1 The student already possesses a collection of isolated ideas of an arithmetical nature.
2 Pure mathematics is an abstract science.

[17] L. E. Boyer, "Preparation of Elementary Arithmetic Teachers," *Emerging Practices in Mathematics Education*, TWENTY-SECOND YEARBOOK of the National Council of Teachers of Mathematics (Washington, D.C.: the National Council, 1957), p. 172.

[18] Wilbur Waggoner, "Improving the Mathematical Competency of Teachers in Training," THE ARITHMETIC TEACHER, V (1958), 84–86.

[19] Grossnickle, *op. cit.*

[20] C. V. Newsom, "Mathematical Background Needed by Teachers of Arithmetic," *The Teaching of Arithmetic*, 50th Yearbook of NSSE, Part II.

3 A truly significant background course in mathematics should demand breadth of study and exploration on the part of the student.
4 The approach of the instructor will be reflected strongly in the ideas put into practice when the student becomes a teacher.

The content recommended by Newsom was as follows:

1 The historical development of mathematics;
2 The real number system;
3 The properties of integers,
   *a*) The four basic arithmetical operations,
   *b*) The fractions,
   *c*) The arithmetic of measurement;
4 Evaluation of formulas;
5 Ratio and proportion;
6 Business arithmetic;
7 Statistical concepts;
8 Probability.

Schaaf,[21] in planning a background course in arithmetic for arithmetic teachers, first took note of Orleans' conclusion that many prospective teachers have difficulty in verbalizing their thoughts when they try to explain arithmetical concepts and processes. With this and the suggestions of Morton, Newsom, and Grossnickle in mind, Schaaf developed a plan for a content course in arithmetic, a skeleton outline of which is presented below:

I. Number Concept and Numeration
   A. Historical Development
   B. Theory of Numeration
II. Nature of Number
   A. Psychological Considerations
   B. Number System of Algebra
   C. Logical Foundations of Arithmetic
III. Computation
   A. Historical Development
   B. Analysis of Theory of Computation
IV. Measurement
   A. Direct
   B. Indirect
   C. Elements of Statistics
V. Socio-Economic Applications
   A. Arithmetic in the Kitchen
   B. Arithmetic in the Market Place
   C. Arithmetic and Finance.

Snader[22] polled a group of specialists in arithmetic concerning the importance of topics for a mathematics course for undergraduates preparing to teach in Grades 1–6. Those which 90 per cent considered important are listed below:

1 Improvement in computation;
2 Increased understanding of number system;
3 Measurement;
4 Concepts of operation;
5 Acquisition of power in reasoning;
6 Postulates of operation (associative, etc.).

Snader also recommended that two years of high-school mathematics should be prerequisite to entrance upon the program in elementary education and that prospective elementary-school teachers should have six semester-hours of content methods in arithmetic.

Stepanowich[23] used a similar procedure with a jury of 70 selected mathematics education specialists in 66 institutions in 32 states. The questionnaire used was concerned with which practices currently employed in college training for prospective elementary-school teachers were favored. The resulting recommendations included:

1 Two years of mathematics should be required for entrance to the elementary-school curriculum.
2 A course in subject matter, separate from the methods course, should be provided.

[21] W. L. Schaaf, "Arithmetic for Arithmetic Teachers," *School Science and Mathematics*, LIII (October, 1953), 537–55.

[22] Daniel Snader, "Mathematical Background for Teachers of Arithmetic," The Arithmetic Teacher, III (March, 1956), 59–65.

[23] Joseph Stepanowich, "The Mathematical Training of Prospective Elementary School Teachers," The Arithmetic Teacher, IV (December, 1957), 240–8.

*a*) This course should be from three to six semester-hours in length.

*b*) The course should be primarily an arithmetic course in which meanings and understandings are emphasized. Related algebra and intuitive geometry should be integrated.

*c*) Teachers preparing to teach in Grades K–6 should be given training separate from teachers preparing to teach in Grades 7 and 8.

Ruddell, Dutton, and Reckzeh[24] reported in the TWENTY-FIFTH YEARBOOK of the National Council of Teachers of Mathematics the results of a questionnaire survey of the professional opinions of the members of the elementary-teacher training departments of the members of the American Association of Colleges for Teacher Education. The questionnaire was organized on the assumption that a six-semester-hour course or sequence would be necessary to provide adequate training. The report includes a number of tables indicating topics and concepts. On the basis of the results of the questionnaire and previous similar findings, the authors recommend a minimum of six semester-hours of background mathematics with emphasis on the development of principles and understanding of concepts.

In a recent article putting forth major issues in the teaching of arithmetic, Madden[25] included the problem of securing adequate teacher competence. As a part of his argument Madden included a statement of the results of a recent poll of college students preparing to teach in the elementary school. Almost all of these young people felt that there should be a basic mathematics course required, as well as a course in teaching methods, with the basic mathematics course especially designed for elementary-school teachers. In discussing the situation, Madden also mentioned the use of the workshop in elementary arithmetic instruction as an in-service education device. He felt that its value depends upon the degree to which students are taught to understand mathematical concepts in their abstract form, although these concepts should be based upon concrete experiences. A workshop in which teachers' activities are confined largely to making A-V materials was described as inadequate.

Two relatively recent experimental studies dealt with just the two procedures discussed by Madden. Knight[26] designed a study to determine the effectiveness of the subject matter of modern mathematics, as compared to traditional mathematics, in the preparation of elementary-school teachers. The subjects consisted of an experimental group of 30 and a control group of 29 taught by the same instructor. The control group was taught a traditional college review course of fundamentals of arithmetic, basic ideas of geometry, and algebra. The experimental group was taught a course in elementary mathematics with the content and spirit of modern mathematics. Material was included on mathematical systems, number systems, mathematical proof, groups, fields, and sets. Pretests and post-tests were designed to measure such abilities as mathematical reasoning, computation skills, and preference for subject areas. Knight concluded that for prospective elementary-school teachers with little mathematical background in high school the subject matter of modern mathematics is more effective than traditional mathematics in developing computational skills.

The study on in-service procedures was one done by Rudd,[27] who attempted to develop procedures likely to promote growth in arithmetic understandings among groups of elementary-school teachers. Fourteen groups (116 teachers) were

[24] Ruddell, Dutton, and Reckzeh, *op. cit.*, p. 306–17.

[25] Richard Madden, "Major Issues in Teaching Arithmetic," *National Elementary Principal*, XXXIX (October 1959), 17–21.

[26] Lyman C. Knight, "A Study of the Effectiveness of the Subject Matter of Modern Mathematics in the Preparation of Elementary School Teachers," Unpublished Ed.D. Thesis, Penn State, 1958.

[27] L. E. Rudd, "Growth of Elementary School Teachers in Arithmetical Understandings Through Inservice Procedures," Unpublished Ph.D. Thesis, Ohio State University, 1957.

given a test of mathematical understandings. Thirty of these teachers participated in an eight-session course devoted to the development of 72 arithmetical understandings. The effectiveness of the course was measured by: (1) a test of basic mathematical understandings, (2) teacher conferences, (3) teacher questionnaires, (4) teacher summaries, (5) classroom observations, and (6) teacher opinion checklists. The essential findings were:

1 Teacher growth in arithmetical understandings may be produced by this means.
2 An inventory of understandings should be taken first.
3 A course for arithmetic teachers is best at the local level in close proximity to the elementary classroom.

Rudd recommended that the instructor of such a course should utilize basic classroom materials and place strong emphasis upon the adaptation and application of materials and procedures of the course to the elementary-school teacher.

There is no research available which indicates that a better comprehension of mathematical concepts on the part of the elementary-school teacher results in better achievement on the part of students. As a matter of fact, the only available research on the subject offers little encouragement. Smail[28] investigated the relation between mean gain in pupil achievement in arithmetic and certain attributes of teachers. His subjects were 97 teachers in Grades 4, 5, and 6 in Sioux Falls, South Dakota. He found significant relationships between:

1 Attitudes toward pupils and pupil mean gain in arithmetic;
2 Number of courses completed in methods of teaching arithmetic and pupil mean gain.

Among combinations for which significant relationships were not indicated were:

1 Teacher understanding of basic mathematical concepts and pupil mean gain;
2 Number of courses completed by teacher in higher mathematics and pupil mean gain.

It would seem that pupil mean gain in arithmetic would be a good criterion measure for the success of arithmetic teaching. There is, of course, nothing in the study quoted which would indicate that it is futile to attempt to improve arithmetic teaching by increasing the mathematical understandings of teachers, but the findings presented should help point up the desirability of exploring the relationship between the teacher's subject-matter knowledge and his pupils' achievement gains. It is also pertinent to note that, with all of the proposed changes in the arithmetic program (as, for example, the program of the University of Illinois Arithmetic Project), those who plan the curriculum in background mathematics for elementary-school teachers should consider that prospective teachers may soon be teaching content which is somewhat different from that currently taught.

Nevertheless, those mathematics educators who have had experience in teaching specialized background courses in arithmetic for elementary-school teachers are enthusiastic about these courses, as witness Mueller:[29]

> When I accepted a position on the mathematics staff of a teachers' college some eight years ago, fresh from teaching the usual mathematics subjects in a liberal arts college, I was amused when I was assigned a course in the content of arithmetic, and remember commenting "How can I teach for a semester on this—what everybody knows?" But I have yet to crowd into the allotted semester all I have wanted to, and I am constantly learning more. The mature and serious study of the content of arithmetic is not only worthy of a place on the college program of a student preparing to enter elementary teaching, but I contend, it represents the most valuable single investment of that student's mathematics-course-taking-time.

[28] Robert W. Smail, "Relationships between Mean Gain in Arithmetic and Certain Attributes of Teachers," Unpublished Ed.D. Thesis, State University of South Dakota, 1959.

[29] Mueller, "Arithmetic and Teacher Preparation," *op. cit.*

### Recommendations

On the basis of the information available, it would seem that certain recommendations for further research are in order:

1 What is the relationship between pupil achievement and teacher knowledge?

Investigations should include experimental studies which take into consideration the specific mathematical knowledge possessed by teachers.

2 What content material and types of presentation provide teachers with the knowledge found most valuable as a result of the above studies?

This is not a proposal for a small-scale local or doctoral study. We don't need any more surveys of what people think should be done or of what schools are presently doing. The need is for long-term investigations with sound theoretical bases and sound experimental designs.

---

# Professional dates

The information below gives the name, date, and place of meeting with the name and address of the person to whom you may write for further information. For information about other meetings, see the previous issues of THE ARITHMETIC TEACHER. Announcements for this column should be sent at least two months prior to the month in which the issue appears to the Executive Secretary, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D.C.

### NCTM Convention dates

*Fortieth Annual Meeting*
April 16–18, 1962
Jack Tar Hotel, San Francisco, California
Kenneth C. Skeen, 3355 Cowell Road, Concord, California

*Joint Meeting with NEA*
July 4, 1962
Denver, Colorado
M. H. Ahrendt, 1201 Sixteenth Street, N.W., Washington 6, D.C.

*Twenty-second Summer Meeting*
August 23–25, 1962
University of Wisconsin, Madison, Wisconsin
H. Van Engen, School of Education, University of Wisconsin, Madison 6, Wisconsin

### Other professional dates

*The Association of Teachers of Mathematics in New England*
December 9, 1962
Wentworth Institute, Boston, Massachusetts
Barbara B. Betts, D. C. Heath and Company, Boston 16, Massachusetts

*Chicago Elementary Teachers Mathematics Club*
December 11, 1961
Private Dining Room. Toffenetti's Restaurant, 65 West Monroe Street, Chicago 1
Mildred C. Rogers, Warren Elementary School, 9210 South Chappel Avenue, Chicago 17.

*Men's Mathematics Club of Chicago and Metropolitan Area*
December 15, 1961
YMCA Hotel, 826 S. Wabash Ave., Chicago 5
Charles W. Moran, 2950 Jarlath Street, Chicago 45, Illinois

*Men's Mathematics Club of Chicago and Metropolitan Area*
January 19, 1962
YMCA Hotel, 826 South Wabash Avenue, Chicago 5, Illinois
Charles W. Moran, 2950 Jarlath Street, Chicago 45, Illinois

*Association of Mathematics Teachers of New Jersey, Sectional Meeting*
January 20, 1962
Jersey City State College, Jersey City, New Jersey
Prof. Ernest R. Duncan, Jersey City State College, Jersey City, New Jersey

*New York Society for the Experimental Study of Education; Section 10—Mathematics*
January 27, 1962
256 Thompson Hall, Teachers College, Columbia University, New York City
Mary G. Rule, 58 Spring Avenue, Bergenfield, New Jersey

*The Mathematics Club of Greater Cincinnati*
February 1, 1962
Shroder Junior High School, 3500 Lumford Place, Cincinnati, Ohio
Roy D. Matthews, Board of Education, 608 East McMillan Street, Cincinnati 6, Ohio

*Association of Mathematics Teachers of New Jersey, Winter Meeting*
February 3, 1962
Paramus High School, Paramus, New Jersey
Prof. Ernest R. Duncan, Jersey City State College, Jersey City, New Jersey

*Women's Mathematics Club of Chicago and Vicinity*
February 3, 1962
English Room, Marshall Field & Company, Chicago 1, Illinois
Dr. Ruth Ballard, University of Illinois, Navy Pier, Chicago 11, Illinois

# Intra-class grouping for arithmetic instruction: critique and criteria

HAROLD H. LERCH
*Southern Illinois University, Carbondale, Illinois*
*Professor Lerch is a member of the elementary education department at Southern Illinois University.*

Attempts to meet the problem of adjusting arithmetic instruction to pupil differences frequently involve the organization of instructional groups of pupils within the classroom. The major purpose of subgrouping in arithmetic classes is to provide for individual differences by bringing pupils of like arithmetical backgrounds, skills, understandings, and concepts together into small groups so that they can study and work effectively at rates and levels that are closer to their own abilities and characteristics. Closely related to the possibility of adjusting instruction to pupil differences are other reasons why intra-class grouping for arithmetic instruction could increase the efficiency of the learning situation or increase the productivity per man-hour of teaching and learning. These reasons generally pertain to the interactions and communications within the class or to the desirability and effects of pupils succeeding at their own levels.

If pupils are working in small groups composed of children with similar needs and abilities, they may find it easier to communicate about their problems with their peers and their teacher. Part of this ease in communication is due to the sharing of similar needs and problems, and part may be due to the reduction of possible interactions. Pupils who are more reluctant to ask questions or to make comments may find the situation more conducive to active participation.

## Ways of grouping

When arithmetic classes are taught as one group, it is not unusual for the questions and comments of a few pupils to take a major portion of the class time. Quite often, the majority of these contributions come from the extreme ends of the ability range, the better or the poorer pupils. In a small group situation, the difficulties of pupils with varying ability levels will be more clearly communicated by word or action to the teacher. Pupils working in small groups should find it easier to ask questions concerning their own difficulties because other pupils in that group will be having similar difficulties. Also, pupils of similar characteristics are not likely to view the questions or comments as immaterial or ridiculous. Because of the reduction in possible interactions and the ease of asking questions, pupils working in small groups with other pupils of similar ability should be able to have a greater amount of personal contact with the teacher. Not only will pupil difficulties become more apparent to the teacher, but procedures and processes may be more clearly communicated to the pupils, and the pupils' skills and understandings may be more closely checked.

When arithmetic classes are subgrouped for instruction, it should be easier for each pupil to work at his own rate and have successful arithmetic experiences at his own level of ability. Success at the pupil's

own level may have a favorable effect upon his attitudes toward arithmetic and lead to further success in the subject. In addition, pupils taught in such a grouping plan may be shifted from group to group according to their own arithmetical needs and abilities. These considerations are not inclusive of all the reasons for intra-class grouping for arithmetic instruction but are indicative of the major arguments.

Several methods of intra-class grouping have been used by teachers intent upon improving arithmetic instruction in their classes. The most prominent among these methods are those involving ability grouping in which standardized test scores, teacher-made test scores, teacher judgments, or combinations of these are used to divide the class into two or three subclasses for arithmetic instruction. In general, these methods have been found wanting and have been severely criticized. The criticisms of intra-class grouping generally pertain to the social stigmas attached to groups, the disruption of class unity, the inflexibility of established groupings, and the inefficiency of criteria used for grouping.

### Criticisms

The most severe criticism of ability grouping in arithmetic classes is the analogy to a caste system in which the lower ability group suffers a loss of self-respect by being referred to as "the dummies." Some critics of ability grouping indicate that children feel the stigma of being assigned to a slow group and that parents tend to look unfavorably upon any plan for instruction which classifies their children as slow learners. The critics imply that in such situations, undesirable attitudes toward the study of arithmetic and the school in general are developed because of the undemocratic social implications involving class distinctions. A review of the research concerned with methods of adjusting arithmetic instruction to pupil differences and their effect upon attitudes toward arithmetic does not verify this particular criticism. It would seem that pupils who do poorly in academic work would have some feelings of inferiority under any organizational plan for arithmetic instruction.

Another criticism of ability grouping for arithmetic instruction is that, in most cases, the unity of the class is disrupted because the subgroups are studying separate topics which may or may not be related. For example, if one group of pupils in a classroom is studying division, another group is studying multiplication, and still another group is studying fractions, the pupils within this room will have few feelings of unity as one arithmetic class because in reality they are three distinct arithmetic classes with a minimum of common bond. It has been suggested that to retain the unity of grouped arithmetic classes, the subgroups should be working on the same topic or area but at different levels of difficulty or abstraction. Under such a plan it would be more possible to bring the entire class together for review sessions or for discussions of particular ideas within a topic.

### Criteria

Although all grouping procedures should provide flexibility in which pupils are moved freely from group to group when circumstances warrant, ability groupings of arithmetic classes in actual practice often have been observed to be quite rigid. The importance of flexibility in grouping for facilitating teaching-learning cannot be overemphasized. Placing a pupil in a group and his participation in that group should not freeze him there. Too often the search for similarity among pupils leads to a situation in which pupils are fitted into a convenient instructional mold which persists for the school year and even indefinitely. An ideal method of intra-class grouping for arithmetic instruction would facilitate the movement of a pupil from one group to another when it became evident that he could work better and to his own greater advantage in another group.

Often the criteria used in the initial grouping restrict the flexibility of the grouped situation. Membership in a group established early in the school year on the bases of standardized test scores or subjective judgments may not change for the entire year. Several considerations may be mentioned concerning the use of standardized test scores and/or subjective judgments as criteria for grouping pupils for arithmetic instruction. Standardized test scores are generally obtained but few times each year, quite often only once a year. If these are to be the bases for establishing groupings, the structure of the classroom groups will tend to be permanent. Some question exists as to whether a pupil's capabilities in one area of arithmetic are the same as those in other areas. Should the ability to do arithmetic be considered as one general ability, or should it be recognized as a complex of specific independent abilities with an underlying aspect of general intelligence? If we recognize there are specific independent abilities, it becomes obvious that using one score from a standardized test as the criterion of grouping for arithmetic is not the most efficient means of establishing the instructional units. Also, within each of the four fundamental processes and the areas to be studied in arithmetic, there are definite levels of difficulty or understanding. Standardized test scores do not point out pupil similarities or differences in levels of achievement within these areas. Thus, they can not be used effectively to group pupils of similar knowledge and understandings for study of particular topics.

Although subjective judgments are quite appropriate for many classroom decisions and situations, some question exists as to whether such judgments are the most valid techniques for this need. Research indicates that subjective judgments of pupils' intelligence and of pupils' achievement in relation to capacity will be incorrect a large percentage of the time. In addition, we are sometimes quite reluctant to alter our initial judgments of pupils' abilities. This could lead to frustration for the pupil who is inaccurately judged. Judgments also are often affected by pupils' personalities or their behavior in the classroom. Such considerations may lead one to infer that subjective judgment is not an effective means of grouping pupils for arithmetic instruction and that more objective data should be used. Yet, subjective judgments based on objective data do have merit. Such objective data regarding pupils' levels of abilities and understandings on particular topics could be obtained from pretests, inventories, or diagnostic tests which pertain specifically to the next topic to be studied.

### More promising methods

Ability grouping, as it has ordinarily been used, does not solve the problem of providing for pupil differences in arithmetic. The mere grouping of pupils with somewhat similar characteristics into instructional units will do little toward improving the teaching-learning situation. Such an organizational plan must be accompanied by the proper selection of content, methods, and materials of instruction suitable to the ability levels of pupils and to the difficulty levels within the topics to be taught. In other words, pupils in the lower ability group should not be struggling with exactly the same content, taught by the same methods, using the same materials as pupils in higher ability groups, and *vice versa*.

Some methods of grouping which have been used successfully in other areas of study do not offer much promise for the teaching of arithmetic. On the other hand, some methods of grouping for reading are similar to effective methods of grouping for arithmetic. Grouping methods which involve the subjective judgment of interests, friendships, or needs generally defeat the purpose of grouping for arithmetic instruction. The use of such criteria does not bring pupils of like backgrounds of understandings, skills, and concepts together into small groups so that they can learn

effectively at their own achievement rates and levels. Instead, pupils in instructional groups thus formed may be quite unlike in their arithmetical backgrounds.

The criticisms of grouping reviewed here are not criticisms of the organizational plan as such but of the various methods of grouping which have been used. As an organizational procedure, intra-class grouping can set the stage for adjusting arithmetic instruction to pupil differences or for improving the teaching-learning situation. The organization does not make the adjustment, but merely establishes an environment in which arithmetic content, teaching methods, and instructional materials can be better adapted to the needs and abilities of pupils. The efficacy of the teaching-learning situation in grouped arithmetic classes will certainly be dependent upon the method or procedure of grouping. On the bases of the previously mentioned criticisms and suggestions, effective methods of intra-class grouping for arithmetic instruction ought to:

1 Use a combination of subjective and objective criteria as the basis for establishing groups.
2 Provide for all levels of ability displayed within the class.
3 Strive to retain class unity.
4 Minimize the loss of self-respect among poorer pupils.
5 Maintain the flexibility that will allow each pupil to work with the group which will most appropriately serve his needs, interests, and abilities.
6 Involve some continuous regrouping procedure in which pupils are grouped and regrouped as new topics, areas, or units are studied and as varying degrees of ability are displayed by pupils.
7 Emphasize the adjustment of content, methods, and materials of instruction to the levels of ability and understanding displayed in a particular group.

---

## Traits of numbers

"Numbers, like men, have character. Just as there are tall men and fat men and cheerful men and honest men, there are even numbers, odd numbers, square numbers, triangular numbers, prime numbers, composite numbers, perfect numbers. Many special traits of numbers have been studied. For example:

Even number—a number divisible by 2
Odd number—a number not divisible by 2
Square number—a number that is the product of another number by itself; $4=2\times2$; $9=3\times3$
Perfect number—a number equal to the sum of all its factors other than the number itself, i.e., $6=1+2+3$."

—Adapted from *The Lore of Large Numbers* by PHILIP J. DAVIS

# Attitudes toward a television program— patterns in arithmetic

GEORGE KAPRELIAN
*Milwaukee Public Schools, Milwaukee, Wisconsin*
*Mr. Kaprelian, a fourth-grade teacher at Walt Whitman School, was a participating teacher in the "Patterns in Arithmetic" pilot study.*

The progress in arithmetic learning of elementary-school children has always been of concern to teachers, parents, and the lay public. The recent crash program in the sciences has led to even greater interest in children's arithmetic achievement. Our colleges and universities have also been demanding a science- and mathematics-orientated student.

The instruments used for measuring the arithmetic competence of children have been, for the most part, various standardized achievement tests. Only during the past few decades, however, has it been recognized that a test of children's attitudes toward arithmetic can be a valuable tool in determining the arithmetic progress of children. There is now reason to believe that the feelings of a pupil toward a subject have a definite bearing on his achievement in the subject.

Dutton,[1] in a study of junior-high-school pupils, found that lack of understanding, difficulty in arithmetic processes, and insecurity in the subject were reasons given by pupils for disliking arithmetic. His study showed that 19 per cent of the pupils expressed extreme dislike for arithmetic, with 5 per cent avoiding the use of arithmetic at all times. Chase[2] reported that arithmetic was second only to language as the most disliked subject by fifth-grade pupils of 65 New England communities. Plank[3] also found dislike for arithmetic in a small group of pupils she studied. These studies, along with others not cited, seem to suggest that there is something to be desired in the arithmetic education of our children.

One of many arithmetic teaching and learning programs formulated with an interest in helping children develop a more favorable attitude toward arithmetic is "Patterns in Arithmetic," a pilot study in arithmetic devised and directed by Dr. Henry Van Engen, Professor of Mathematics and Education, University of Wisconsin, for the Wisconsin Improvement Program. The program is part of a three-year study to investigate whether a new approach to arithmetic teaching and learning might produce greater pupil understanding of arithmetical processes.

The "Patterns in Arithmetic" program includes two television presentations weekly of fifteen-minute sessions to the children and classroom teachers; arithmetic work

[1] Wilbur H. Dutton, "Attitudes of Junior High School Pupils Toward Arithmetic," *School Review* (January, 1956), p. 18.

[2] W. Linwood Chase, "Subject Preferences of Fifth-Grade Children," *Elementary School Journal* (December, 1949), p. 204.

[3] Emma R. Plank, "Observations on Attitudes of Young Children Toward Mathematics," THE MATHEMATICS TEACHER, XLIII (October, 1950), 252.

sheets to be completed by the children after each telecast, and periodic workshop meetings of classroom teachers with the director and the television teacher.

The purpose of this investigation is to observe the feelings and attitudes of 4A children at Walt Whitman School toward a new approach in arithmetical thinking and working. In general, the questions were directed toward obtaining pupil reaction to three main questions:

1 Were the children developing favorable attitudes toward an academic procedure which was, for the most part, in diverse contrast to their learning from the previous four years of schooling in the particular area of study?
2 What effect was television playing on the importance of the role of the individual classroom teacher in the pupils' arithmetic instruction?
3 Did the pupils notice any growth in the arithmetic processes; if so, which specific ones?

### Procedure for investigation

A coded questionnaire was administered to three classes including sixty-five 4A pupils during three sittings by the same teacher. Pupils were requested not to write their names on the questionnaire sheet. Time was devoted to informing the children that only the over-all reaction to "Patterns in Arithmetic" was being sought, not the feeling of particular individuals.

Each question was read to the pupils by the teacher, and all were asked to respond. All requests for reinterpretation or elaboration on questions asked in the questionnaire were granted. During the period for responding to questions, the teacher made comments to reassure the subjects that the group reaction to "Patterns in Arithmetic" was the only objective desired from the pupils' responses to the questionnaire. It was hoped that such an approach would promote an expression of the inner thoughts and feelings of the 4A children.

### Interpretation of data

The data gathered from the questionnaire is presented in the form of various graphic illustrations.

The interpretation of the data includes the subjective analysis of the factors which might have contributed to the attitudes of the pupils. Comments made by children, scores made by pupils on work sheets, and general attitudes toward arithmetic as a subject in the class program were observed and studied.

Figure 1 interprets the results of the question, "How much do you like our new arithmetic?" The children were offered the

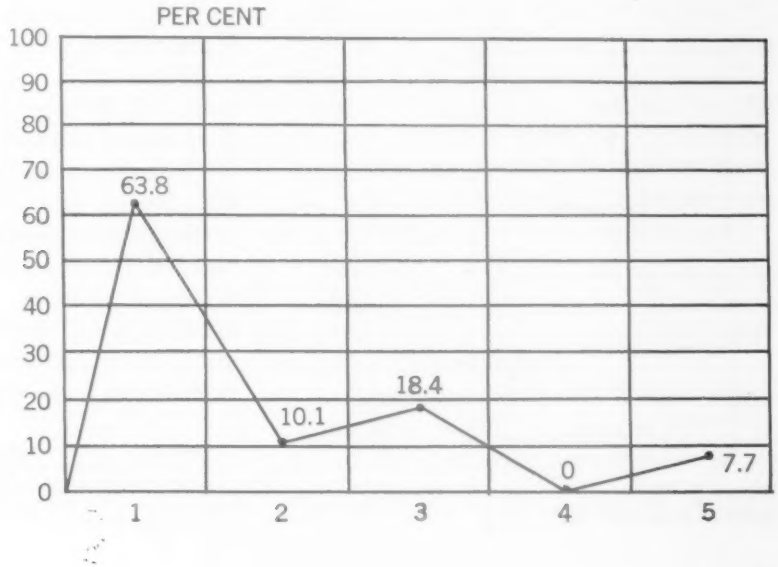



*Figure 1. How much do you like our new arithmetic?*

following optional answers: (1) very much; (2) much; (3) some; (4) very little; (5) dislike it. The data seem to suggest that the pupils favorably accepted the program of "Patterns in Arithmetic." It is interesting to note that 92.3 per cent of the children have expressed some form of approval of the program. The lack of selection of option 4 by pupils could have been the result of too broad a range in the particular question.

The favorable attitude of pupils toward the new program might be due to several reasons. Pupils seem to feel comfortable in the processes of division (Fig. 2) and prose tasks (Fig. 3). There is reason to believe that success or failure in these arithmetic tasks would play an important part in pupil reaction to the subject. Figures 4 and 5 indicate the contributions of the classroom teacher and television to the attitudes of children toward the program.

The selection of division as the favorite arithmetical process learned in the program is a surprise. Many pupils were fascinated with the new frame which is used in division; yet, they are confused by the freedom permitted in the partial quotients. Estimation of the partial quotients is still a major handicap in the division process.

The rank position of rate pairs among the tasks in Figure 2 is probably due to the

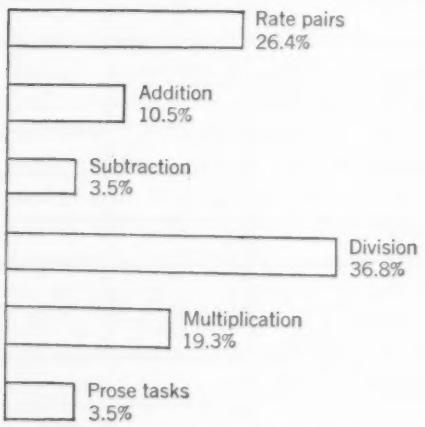



*Figure 2. "What part or parts of our new arithmetic do you like?"*

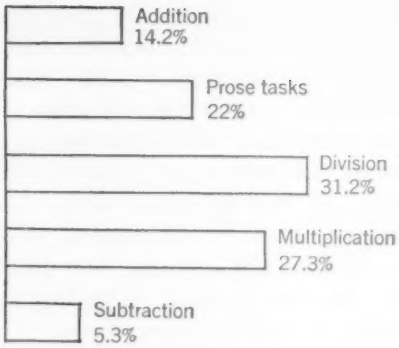



*Figure 3. "In which arithmetic process has our new arithmetic helped you most?"*

recent presentation of the process in the program. The achievement of the youngsters in the process also would have some bearing on its position among the tasks.

The selection of addition by 10.5 per cent of the 4A's might be due to the method of mental computation employed in teaching the process. Multiplication could have been selected because of the use of the distributive law, which enabled the children to multiply by two-digit numbers. Mental computation was also used in multiplication. These procedures stimulated pupil interest.

The interest in prose tasks of the children is rather puzzling. Only 3.5 per cent of the 4A's chose it as their favorite arithmetic process in the program; yet, as shown in Figure 3, 22 per cent of the 4A's said that the arithmetic program helped them most in prose tasks.

The selection of prose tasks, multiplication, and division as the three processes that helped children most in the program is most encouraging. It is especially interesting to observe the high rating for prose tasks, since recent studies by Collier[4] and Fulkerson[5] reported noticeable weaknesses of children and prospective elementary-school teachers in problem solving. *Use of equations in problem solving might be helpful in improving the attitudes as*

[4] Calhoun C. Collier, "Blocks to Arithmetical Understanding," THE ARITHMETIC TEACHER, VI (November, 1959), 264.
[5] Elbert Fulkerson, "How Well Do 158 Prospective Elementary Teachers Know Arithmetic?" THE ARITHMETIC TEACHER, VII (March, 1960), 141–6.

*well as the competency of people in problem solving.*

The responses of the 4A pupils on their reaction to television as a teaching aid in arithmetic instruction seem to be quite apparent. The unknown factor in the 69 per cent who felt that television arithmetic helped them very much would be the part played by the teacher. (See Fig. 4.) If, as indicated in Figure 5, the children believe that they would need more teacher aid when arithmetic is presented from television, then the interest and ability of the classroom teacher become more vital in the learning process.

Of the 3.2 per cent pupils of 4A groups at Whitman School, who believe that the new arithmetic program mixes them up, one child's situation introduced a problem which might be difficult to handle in such a program. This child began as a participant in the program at its introduction, moved out of the school district after two months, and returned to the same school seven weeks later. Whether children can adjust to such a program after missing numerous arithmetic experiences from television is a problem that will need to be faced and resolved.

The question of the role of the teacher in learning from television is a key discussion issue among educators in our public schools. One viewpoint is that educational television may someday replace the classroom teacher, while the other argument is that the classroom teacher still will be a major party in the educational process. Figure 5 shows the feelings of the pupils toward the role of their teacher in the "Patterns in Arithmetic" program. There could be several reasons for needing more teacher aid when arithmetic is presented via television.

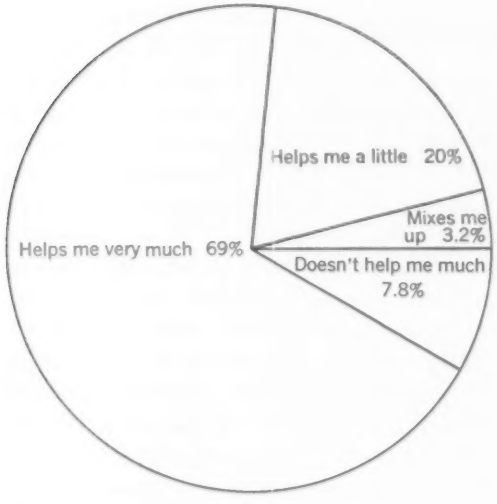



*Figure 4. "How much help is television arithmetic for you?"*

| When it is taught from television | When it is taught from a book by the teacher |
|---|---|
| 55.5% | 44.5% |

*Figure 5. "When do you need the most help in arithmetic?"*

The duties of the classroom teacher in a television instructional situation vary considerably from those of the usual teaching requirements. During the television arithmetic presentation the teacher must be alert and sensitive to all aspects of the situation. The attention, responses, and work plan for the day must be handled without delay. In addition, the teacher should have a thorough knowledge of the ideas being presented in the program. Also, the teacher needs to be aware of the degree to which children are having difficulty in comprehending the presentation from the program.

Figure 6 graphs the answers to the question, "How has the new arithmetic helped change your feelings about number work or arithmetic?" The children had a choice of the following responses:

1 Not much, because I always liked arithmetic.
2 I like it better now because I can understand arithmetic better.
3 I didn't like arithmetic before, and I still don't like it.
4 I dislike arithmetic more now.

The results are very encouraging. Granted,

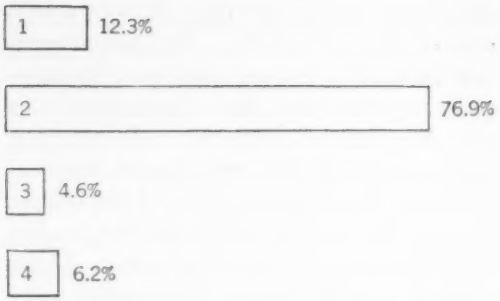



*Figure 6. How has the new arithmetic helped change your feelings about arithmetic?*

the inclusion of an additional option might have changed the over-all results somewhat, but one still needs to look with interest at option 2, which indicates that *over three-fourths of the pupils have taken a more favorable attitude toward arithmetic because the program has helped their understanding of the subject.* The results of this question compliment the total plan, the presentation, and the personnel in this program.

**Summary and conclusions**

The television program, "Patterns in Arithmetic," viewed by sixty-five 4A pupils at Walt Whitman School, has been favorably accepted by the pupils. The children have reacted favorably to the method by which the program is presented. There also is sufficient evidence to suggest that the program has helped in the creating of a more favorable attitude toward arithmetic as a subject.

The questionnaire data indicates that the arithmetic program has been helpful to children in five arithmetic processes. Division, multiplication, and prose tasks are believed to be the arithmetic skills in which the program of "Patterns in Arithmetic" has been most helpful.

The pupils of Walt Whitman School in the "Patterns in Arithmetic" project feel that they need more teacher aid and direction when the subject is presented from television than when it is learned in the conventional classroom situation.

---

## Can machines think?

"It all depends on what you mean by 'think.' Computers can calculate, remember, compare, and correct themselves, which is more than a lot of people can do. But it is unlikely that computers will ever come up with a totally new concept—that outburst of sheer creative imagination."—From "Special Report from United States Steel," *Time*, Vol. LXXVIII, October 20, 1961.

# E pluribus unum—
# a brief discussion on the "law of one"

JOHN K. RECKZEH AND ERNEST R. DUNCAN

*Jersey City State College, Jersey City, New Jersey*

*Professors Reckzeh and Duncan are members of the department of mathematics at Jersey City State College.*

Next to discovering something new, there is probably nothing that pleases a mathematician as much as discovering that an apparently obvious fact is of fundamental significance.

The fact that the product of one and any number is equal to that number seems obvious to most people. But this apparently trivial property of the number one is important and useful in mathematics. Diophantus drew attention to it centuries ago, and modern mathematicians acknowledge its significance today.

The importance of the above property of one is that it is an instance of the *identity element*, a fundamental concept in modern abstract algebra. If $a \cdot b$ indicates that the mathematical operation $\cdot$ is being performed on two elements of a set, $a$ and $b$, then $a$ is the identity element if $a \cdot b = b$ for every $b$ in the set. Thus, with the set of numbers, one is the identity element with respect to multiplication and zero is the identity element with respect to addition. For $0+y=y$ and $1 \times y = y$, if $y$ is a number.

The significance that mathematicians attach to the identity element is shown by the fact that when they encounter a new set of elements with a new mathematical operation, one of their first efforts will be a search for an identity element.

It is the purpose of this article to demonstrate some of the many places in everyday arithmetic where an understanding of the identity property of one will be valuable. For convenience this property will be referred to as the *Law of One.*

Before discussing examples of the application of the Law of One, however, it is necessary to consider another idea, for the Law of One is of little value unless this additional idea is understood. This idea is that while there is one and only one number "one," there are many symbols for expressing this number—or as the current literature frequently states it, there are many names for the number one.

Some of the unlimited number of names for the number one are:

$$100\%; \quad \frac{2}{2}; \quad 4 \times \frac{1}{4}; \quad \frac{1}{2}+\frac{1}{2}; \quad \frac{x}{x}\{x \neq 0\}.$$

The two ideas, the Law of One and the fact that the number one has many names, together provide a simple explanation for a rule used almost universally in the teaching of arithmetic: the value of a fraction is not changed if numerator and denominator are both multiplied by any number, except zero.

This rule is strongly entrenched in the traditional teaching of arithmetic and algebra. No plea is made here to eliminate the rule, but the authors do firmly believe that it is an error not to connect it with the Law of One as follows:

$$a) \quad \frac{2}{3}=\frac{2}{3} \times 1=\frac{2}{3} \times \frac{2}{2}=\frac{4}{6};$$

$$b)\quad \frac{21}{35}=\frac{21}{35}\times 1=\frac{21}{35}\times\frac{\frac{1}{7}}{\frac{1}{7}}=\frac{3}{5};$$

$$c)\quad \frac{a}{b}=\frac{a}{b}\times 1=\frac{a}{b}\times\frac{x}{x}=\frac{ax}{bx}\qquad \{x\neq 0\};$$

$$d)\quad \frac{2}{\sqrt{2}}=\frac{2}{\sqrt{2}}\times 1=\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{2\sqrt{2}}{2}=\sqrt{2};$$

$$e)\quad \frac{3}{-5}=\frac{3}{-5}\times 1=\frac{3}{-5}\times\frac{-1}{-1}=\frac{-3}{5}.$$

The second step in each of the above examples is not necessary and will be eliminated in future examples.

The example *a*) above uses the Law of One to illustrate that $\frac{2}{3}$ and $\frac{4}{6}$ are different names for the same number. A similar statement may be made for each of the other examples above.

The Law of One may be used to explain certain short cuts as follows:

$$a)\quad 25\times 88=(25\times 88)(4\times\tfrac{1}{4})$$
$$=(4\times 25)(\tfrac{1}{4}\times 88)=2200;$$

$$b)\quad 16\tfrac{2}{3}\times 78=(16\tfrac{2}{3}\times 78)(6\times\tfrac{1}{6})$$
$$=(6\times 16\tfrac{2}{3})(\tfrac{1}{6}\times 78)=1300.$$

This method is particularly suitable for aliquot parts. An aliquot part of 100 is contained in 100 an integral number of times. Since $16\frac{2}{3}$ is contained six times in 100, it is an aliquot part of 100.

The Law of One also may be used in conjunction with the Distributive Law to add fractions:

$$\tfrac{1}{2}+\tfrac{1}{3}=(\tfrac{1}{2}+\tfrac{1}{3})(6\times\tfrac{1}{6})=[6(\tfrac{1}{2}+\tfrac{1}{3})]\times\tfrac{1}{6}$$
$$=(3+2)\tfrac{1}{6}=\tfrac{5}{6}.$$

The Law of One is helpful in changing to per cent:

*a*) $\frac{1}{4}=\frac{1}{4}(100\%)=25\%$;

*b*) $\frac{1}{6}=\frac{1}{6}(100\%)=16\frac{2}{3}\%$.

It is recommended by many that "invert and multiply" be eliminated by using the Law of One in the following manner:

$$\frac{\frac{2}{3}}{\frac{3}{4}}=\frac{\frac{2}{3}}{\frac{3}{4}}\times\frac{12}{12}=\frac{8}{9}.$$

If one still wishes to "invert and multiply," this process may be justified in the following manner:

$$\frac{2}{3}\div\frac{3}{4}=\frac{\frac{2}{3}}{\frac{3}{4}}\times\frac{\frac{4}{3}}{\frac{4}{3}}=\frac{\frac{2}{3}\times\frac{4}{3}}{1}=\frac{2}{3}\times\frac{4}{3};$$

or, in general,

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{\frac{a}{b}}{\frac{c}{d}}\cdot\frac{\frac{d}{c}}{\frac{d}{c}}=\frac{\frac{a}{b}\times\frac{d}{c}}{1}=\frac{a}{b}\times\frac{d}{c}.$$

In long division the standard practice is to replace the division $2.31\overline{)7.3124}$ by $231\overline{)731.24}$. This is readily explained by the Law of One as follows:

$$2.31\overline{)7.3124}=\frac{7.3124}{2.31},$$

$$\frac{7.3124}{2.31}=\frac{7.3124}{2.31}\times\frac{100}{100}=\frac{731.24}{231}.$$

Scientific notation may also be explained in terms of the Law of One:

$$93{,}000{,}000=93{,}000{,}000\times 1$$
$$=93{,}000{,}000\times\frac{10{,}000{,}000}{10{,}000{,}000}$$
$$=\frac{93{,}000{,}000}{10{,}000{,}000}\times 10{,}000{,}000$$
$$=9.3\times 10^{7}.$$

If negative exponents are available, then new expressions (names) for one are available, such as:

$1=10\times 10^{-1}$, $1=10^{3}\times 10^{-3}$, $1=10^{7}\times 10^{-7}$.

Then:

$$93{,}000{,}000=93{,}000{,}000\times(10^{-7}\times 10^{7})$$
$$=(93{,}000{,}000\times 10^{-7})\times 10^{7}$$
$$=9.3\times 10^{7},$$

and

$$.00031 = .00031 \times (10^4 \times 10^{-4})$$
$$= (.00031 \times 10^4) \times 10^{-4}$$
$$= 3.1 \times 10^{-4}.$$

Many other uses of the Law of One might be listed. The discovery of fresh applications is a valuable exercise for students, since this law emphasizes one of the basic concepts of modern mathematical structure.

It should be emphasized again that the "Law of One" is a convenient expression used in this article. In standard mathematical usage, the number one is referred to as the identity element with respect to multiplication, or as the multiplicative identity.

When arithmetic is taught using the Law of One as illustrated in this article, several advantages are obtained:

1 Fewer rules are necessary and the rules used are more basic. For example, "Invert and Multiply" can be eliminated.
2 Better explanations may be given for rules. See the example on division of decimals or that on multiplying or dividing the numerator and denominator of a fraction.
3 Progress in algebra is directly related to understanding of arithmetic. The identity property of the number one is probably more useful in algebra than in arithmetic. Emphasizing this property in arithmetic provides a better foundation for algebra.

---

## Place value

In spite of day-in, day-out contact with it, place value is not understood. Here is a game that is easy and fun, and it brings home the idea that where you place the number determines its value.

This game can be extended to more places, but we recommend keeping it in groups of three to emphasize that no matter how large a number we read, we read it in groups of three. 999 is really the largest number we are called upon to read.

| *You* | *Your opponent* |
|---|---|
| – – – | – – – |

Write a three-place number but keep it hidden. Your opponent does the same. Take turns guessing which digit is in which place. The first one to complete the numeral is the winner.

You should ask your opponent, "Is there a three?" If he says, "No," it is his turn. If he says "Yes," it is still your turn and you ask, "Is it in hundreds (or tens or ones) place?" If your opponent says, "Yes," you call it 300 and write the numeral in its place, 3 – –. It is your turn as long as your opponent says yes.—MARIE LUTZ, *Syracuse 8, New York*

# Discovering bar graphs by degrees

THOMAS J. JENNINGS, *Garden City, New York*

*Mr. Jennings teaches sixth grade in Stratford Avenue School in Garden City, New York, and is a member of the part-time faculty at Hofstra College, Hempstead, New York. He is particularly interested in elementary-school mathematics.*

Quite frequently youngsters are capable of learning a great deal more than we suspect, if they become actively involved in a discovery approach to the subject. A heterogeneous class of children who had just finished third grade helped illustrate this belief.

It was decided that we would keep a daily record of the temperature. The class was given a chance to become familiar with two types of thermometers, one with a vertical scale, and one with a semicircular scale. Each morning, at the same time, two different volunteers were given the opportunity to read the thermometers and compare readings. Usually the two readings were identical. On one of the occasions that the readings differed, one was 82°, and the other was 80°. Without hesitation, the class chose 81° as the temperature to be recorded. They had done averaging intuitively.

After the reading was taken, a model thermometer was brought forth for the class to study. They were urged to find how to record the temperature on this model thermometer. A volunteer readily set the ribbon to correspond with the reading for the day. At this point, the model thermometer was put aside until the next day.

The procedure of temperature taking was followed the next day. When it came time to reset the model thermometer to the new reading, the class was guided to see there would be no record of the previous day's temperature. Some method had to be worked out to keep a permanent record of the readings.

It was suggested that the record be kept on the blackboard or on sheets of paper, but these ideas were put aside in favor of writing the daily temperature in the boxes of a large, oaktag calendar which was prominently displayed on the wall.

For the next several days the readings were taken, set on the model thermometer, and listed in the appropriate box on the wall calendar. To highlight the awkwardness of reading the record from the calendar, the class was asked, "Which was the hottest day?" "Which was the coolest?" "On which two days was the reading the same?" It was immediately apparent that finding the answers from the calendar was difficult. The class was asked to think of a better way to keep the record.

The next day the usual procedure was followed. This time, however, a large, oaktag chart was on display in front of the room. On this chart was a full-sized reproduction of the scale of the model thermometer, though no numbers were included.

Several youngsters noticed the similarity between the chart and the scale on the model thermometer. At their suggestion, the appropriate numbers were added to the chart, and the class was challenged to find a way of making a "picture" of the day's reading on the wall chart.

After several minutes of questions and answers, one youngster suggested match-

ing up the model thermometer with the wall chart so their numbers corresponded. A ruler was used to draw a line from the reading on the model thermometer to the wall chart. This line was readily converted to a red bar to complete the "picture" of the ribbon on the model thermometer. It was a simple task to guide the class to discover the need for putting a date under each bar. Now, it was no longer necessary to list the daily temperature reading on the wall calendar.

Following the usual procedure, the next day brought the class face to face with the problem of where to draw the next bar. It was readily seen that putting the second bar right beside the first one made them difficult to distinguish. It was decided that some space should be left between each bar.

The next step was to encourage the class to find a way to record the temperature on the chart without having to place the model thermometer against it. It did not take long for them to discover that a bar could be drawn on the chart to correspond to the setting on the model thermometer.

At this time, each youngster was given a sheet of graph paper, though it was not called by that name. Using this squared paper, they were asked to find a way to make a "picture" of the wall chart.

It took several days before a workable answer was advanced, but it represented careful thought. Each class member was able to make his own personal graph. They took great pride in recording the daily temperature reading.

It was only when they reached this stage that the class was told they had been working with bar graphs. This development can be done by using only fifteen or twenty minutes of the daily arithmetic lesson; therefore, other topics may be covered at the same time.

The use of such an introduction to bar graphs seems to offer a number of desirable experiences to the class:

1 By choosing different volunteers each day, a great many youngsters become actively involved in the lessons.
2 Youngsters become familiar with the handling and reading of vertical-scale and semicircular-scale thermometers.
3 The understanding of why the readings must be taken at the same time each day contributes to the development of a scientific attitude.
4 When thermometer readings differ, the youngsters intuitively average them.
5 The keeping of records and the labeling of charts encourage good work habits.
6 The class develops the understanding that bar graphs are tools.

Most important of all, youngsters find that it's exciting to think.

## Books received

*Enlarging Mathematical Ideas*, Ginn Arithmetic Enrichment Program for Grade 5, John L. Marks, James R. Smart, and Irene Sauble. Boston: Ginn & Company, 1961. Paper, 96 pp., $0.64.

*Exploring Mathematics on Your Own*, Series of nine pamphlets, Donovan A. Johnson and William H. Glenn. St. Louis 26: Webster Publishing Company, 1961. (Pamphlet titles: Sets, Sentences, and Operations; The Pythagorean Theorem; Topology; Understanding Numeration Systems; Fun with Mathematics; Number Patterns; Invitation to Mathematics; The World of Measurement; Adventures in Graphing.) Set, $9.54.

*Improving Mathematics Programs*, Editor, M. Vere DeVault. Columbus, Ohio: Charles E. Merrill Books, Inc., 1961. Cloth, 552 pp., $7.95.

*The Lore of Large Numbers*, Philip J. Davis. New York: Random House, 1961. Paper, 165 pp., $1.95.

*New Thinking in School Mathematics*, Organisation for European Economic Co-operation. Washington 6, D.C.: O.E.E.C. Mission, 1346 Connecticut Ave., N.W., 1961. Paper, 246 pp., $2.50.

*Numbers for You and Me*, Dan T. Dawson. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1961. Paper, 96 pp., $0.88.

*The Story of Numbers*, Patricia Lauber. New York: Random House, 1961. Cloth, 80 pp., $1.95.

# Evaluation of a mathematics program

GEORGE F. MADAUS

*Worcester Public Schools, Worcester, Massachusetts*

*Mr. Madaus is a member of the mathematics department at Forest Grove Junior High School.*

Mathematics, with its content, nature, scope, and instruction has been during recent years sharply focused in the eye of the American public. The mathematics curriculum in the schools has undergone, or is about to undergo, major revisions.

The public mistakenly believes this is a post-Sputnik development. In part due to the Russian accomplishments, the revision of the curriculum, the retraining of teachers, and the writing of new textbooks have been accelerated. However, this change predates the space race by many years.

Regardless of when it started, this change is now fully upon us. As a current teacher of mathematics, evaluation of my pupils' progress, in the light of the new mathematics, is a daily task, one which must be done expertly, accurately, and with dispatch.

The mathematics teacher can no longer evaluate merely computational skills. His evaluation, like the subject itself, is not a departmentalized or fragmentary thing but a unified whole. Thus, good evaluation must take into account the triple nature of mathematics, that of a tool, langauge, and mode of thought.[1]

First, and probably most important, good evaluation leads to improved instruction.[2] Evaluation activities indicate the effectiveness of instruction techniques, choice of materials, and assignments. These evaluation techniques immediately point out what has been understood by the student, and what has been missed, data needing re-emphasis, or perhaps the need of an entirely new approach. Thus evaluation, when properly used, gives direction and guidance to daily instructional planning and development.

Second, good evaluation improves the progress of individual students. Tests and other appraisal devices can determine the pupil's readiness for future instruction. They can diagnose individual weaknesses and strengths. These measures can, when used properly, locate difficulties and faulty mathematical reasoning processes.[3] If a classroom teacher selects from the above uses of evaluation techniques, he can take care of individual differences and programs. The weak can be better helped while the more mature mathematical minds can be enriched and their program of studies accelerated.

Third, good evaluation can be used to answer many questions the administration or supervisory staff may have concerning the mathematics curriculum. If properly used, evaluatory techniques bring to the fore the strengths and weaknesses of the total school's mathematics program. These same techniques, in the hands of competent personnel, aid in the proper selection of textbooks to meet the new needs of modern mathematics.

Fourth, evaluation activities create competition among the students, with one's own record, the class record, or even

[1] Stewart Cairns, "Mathematical Education and the Scientific Revolution," THE MATHEMATICS TEACHER, LIII (February, 1960), 66.

[2] Donovan A. Johnson, "What Can the Classroom Teacher Do About Evaluation?" *The Bulletin of the National Association of Secondary-School Principals*, XLIII (May, 1959), 155.

[3] *Ibid.*

with regional or national norms. This in itself can be a stimulating experience, as most students are proud of a job well done. "Completing a test is an intense learning experience in itself. Discussing and correcting errors on a test help to locate errors, misunderstandings, and wrong procedures, as well as reinforce correct learning."[4]

These, then, are reasons for evaluation. Evaluation obviously serves multipurpose ends. These ends, and perhaps others could be listed by other teachers, justify the time and energy spent by the classroom teacher in arriving at a good, workable, and practical evaluatory program.

### Goals to be evaluated

In order to have direction and coherence in his evaluation, the classroom teacher must have clearly in his mind, before starting to evaluate, the goals or objectives of his instruction. Only by clearly knowing our goals can we evaluate and measure progress, or lack of it, toward their attainment.

The goals or objectives stated in a course of studies for a school system, by the administration, will, of course, affect the goals of individual teachers to a greater or lesser extent. The way the administration conducts school-wide testing programs will greatly influence individual teachers' goals, depending on the teachers' interpretations of the reasons behind the issuance of the tests.[5]

It is obvious, then, that goals vary from teacher to teacher and from system to system.[6] However, certain goals in mathematics instruction are immutable in the light of the new mathematics. These basic goals, that must be evaluated daily, consist of the following:

1 Does the student show growth and understanding with regard to basic mathematical concepts? That is, does he recognize and understand the underlying patterns and structure of mathematics so that he can see these patterns and structure in any problem which confronts him?

2 Does the student possess the necessary skills in performing mathematical operations and processes?[7]

3 Is the student accurate and precise? As mathematics is a precise and exact science, the qualities of accuracy and precision should be fostered in the pupils.[8]

4 Does the pupil have an enthusiasm for and enjoyment of mathematics? This "feeling" for mathematics, even to the extent of a love of the subject, should be instilled in every pupil. The extent to which this can be accomplished depends in large measure on the instructor's feeling for the subject. Nevertheless, it should be a goal of every classroom instructor.

5 Does the child see the place of mathematics in our complex society? Does he know who the prominent mathematicians are and what they do? This goal, while largely neglected, is of extreme importance, for children emulate idols whom they know well. Thus, a child might strive to be a doctor because of the work of Dr. Salk or Dr. White. Almost every eighth grader knows who Bob Cousy is, but only a scant handful of adults can give you the names and accomplishments of *only* three prominent mathematicians whose work has shaped the destiny of modern society.

6 Is the child able to use comfortably the unique language and symbolism of mathematics?

While goals of individual sections of mathematical instruction are specific (for example, the goals in teaching understanding of the decimal system are very spe-

[4] *Ibid.*

[5] Laura K. Eads, "Evaluation of Learning in Arithmetic," *The Bulletin of the National Association of Secondary-School Principals*, XLIII (May, 1959), 129.

[6] *Ibid.*

[7] John J. Kinsella, "Evaluation of Student Learning in Secondary Schools," *The Bulletin of the National Association of Secondary-School Principals*, XLIII (May, 1959), 125.

[8] Marian C. Cliffe, "Mathematics Evaluation in a Large City," *The Bulletin of the National Association of Secondary-School Principals*, XLIII (May, 1959), 162.

cific), the foregoing six listed are the general principles or goals that govern one's choice of particular goals.

### Types of evaluatory techniques

So far, the purposes of evaluation and the goals to be evaluated have been discussed. We must now examine the techniques used to evaluate the aforementioned goals.

Since our goals are varied, it is immediately apparent that to be successful we shall need to use a variety of evaluation instruments in our classroom.[9]

The first and main type of evaluation instrument is, of course, the teacher-made test or examination. The first type of teacher-made mathematics test, and I hasten to add, the least important, is a test of computational skills and problems. A typical item on this type of test might be:

(1) Multiply and check:

(a) $132 \times 457$

or;

(2) 23% of 160 is ____?

These items merely test computational ability. There is, of course, still a strong place for such items in our testing in light of our second goal. One must be able to compute to do mathematics.

More important, however, than testing computational ability is the teacher-made test which examines meanings and understandings.[10] The fact that a child gives the correct answer to the computation $25+38$ indicates that he knows how to obtain the correct answer, but it does not necessarily indicate that he understands what he did to obtain the correct answer.[11] However, if the child could successfully do this type of item:

3 hundreds and 4 tens and 6 ones
+2 hundreds and 6 tens and 5 ones
___ hundreds and _ tens and _ ones or
_____,

it would indicate he not only knows how to add, but also understands what he is doing.

It is extremely important that a teacher frame his tests to measure understanding. Many teachers believe this to be a difficult procedure. Quite the contrary, construction of items to test understanding, while requiring thought, is not difficult. Here are two illustrations of items which test for understanding:

1 Group the x's shown below with lines to show their meaning in base four:

x x x x x x

x x x x x x

x x x x x x

2 In the numeral below in base ten, which of the 2's stands for two thousand?[12]

*a b c*
21232

(1) *a*
(2) *b*
(3) *c*
(4) none of the above ( )

Teacher-made tests should also test the pupil's ability to read mathematical symbols and vocabulary. Mathematics has a language all its own. If a pupil does not understand its language, he is unable to do mathematics because he loses himself in the symbolism. Typical items to test this skill might be:

[9] Johnson, *op. cit.*, p. 156.
[10] Vincent J. Glennon, "Evaluation in Arithmetic and Talented Students," *The Bulletin of the National Association of Secondary-School Principals*, XLIII (May, 1959), 134.
[11] Robert H. Koenker, "Measuring the Meaning of Arithmetic," THE ARITHMETIC TEACHER, VII (February, 1960), 93.
[12] School Mathematics Study Group, *Junior High School Mathematics Units*, Volume I, *Number Systems, Teachers' Commentary* (New Haven, Connecticut: Yale University Press, 1959).

(1) Insert one of the following symbols ($<$, $>$, $=$) which will make the statements below correct.

($a$) $(33\times t)_{12}$ _____ $(t\times 33)_{12}$

($b$) $625-25$ _____ $25-625$

(2) Which of the following are prime numbers?[13]

($a$) 1; ($b$) 2; ($c$) 51; ($d$) 91; ($e$) 83

($A$) $a$, $b$, and $e$ are primes.

($B$) $a$, $b$, $d$, and $e$ are primes.

($C$) $b$ and $e$ are primes.

($D$) all of the above are primes.

( )

Teachers should also frame their tests to measure attitudes, logical analysis in problem solving, and logic. Other types of tests the teacher may use to evaluate are the open-book tests and the take-home tests.[14]

Since testing by the teacher is a key evaluatory technique, care, thought, and skill should be used by the teachers when constructing their tests.

"The most penetrating evaluation technique available to the teacher is the observation-interview. It is generally agreed that this is the only technique that enables the teacher to get at the thought processes being used by the learner."[15] Written tests provide inadequate evaluation of my pupils' achievements. Daily interviews in the form of oral questions to the pupils and their answers, the observation of class discussion about points in question, and the questions which the students themselves ask help in evaluating the level of maturity and the grasp which the student has of many mathematical patterns.

One can evaluate a pupil's enthusiasm for, and appreciation of, mathematics by the caliber of his work on a special assignment. These assignments might include a paper on the history of numbers, a biography of a famous mathematician, or a paper discussing the workings of the binary system.

Curiosity and creativeness can be measured by the pupil's ability to handle an assignment such as this: Prove that the sum of two odd numbers is always an even number.

In this type of assignment or in the special project assignment, a teacher can evaluate a child's skill in communicating mathematically, his craftsmanship in the use of mathematical principles, his organization of mathematical properties, and his mathematical maturity and creativeness.[16]

Besides teacher-made appraisal instruments, there is a wealth of published evaluation materials. "The standardized test is useful for the evaluation of talented students in computational skills and verbal problem-solving abilities."[17] There are excellent diagnostic and prognostic tests constructed by experts to help us evaluate a pupil's work.

While nothing can replace the teacher-made test in evaluating specific goals of learning, the good teacher should familiarize himself with any standardized commercial evaluating instrument.

### Summary

The above, then, are the various techniques, instruments, and measures available to a classroom instructor of mathematics. Their judicious use will insure correct objective evaluation of the teacher's students.

Evaluation is an important phase of any good modern mathematics program. To evaluate accurately, the teacher must know why he is evaluating, what he is evaluating, and how he can best evaluate that which requires grading.

[13] *Ibid.*, p. 31.
[14] Johnson, *op. cit.*, p. 156.
[15] Glennon, *op. cit.*, p. 135.
[16] Johnson, *op. cit.*, p. 160.
[17] Glennon, *op. cit.*, p. 134.

# San Diego's new arithmetic program

J. FRED WILSON, WILLIAM SELF, AND DON CHAMBERLIN

*San Diego City Schools, San Diego, California*

*Mr. Wilson is a director of elementary education for the San Diego City Schools; Mr. Self, an elementary principal; Mr. Chamberlin, an elementary supervisor.*

The curriculum challenge of the 1960's is unquestionably the development of new mathematical material. The problems of bridging the gap between the traditional arithmetic content and methodology to the new and challenging modern mathematics approach will test the ingenuity of all concerned throughout this decade. What are the values of the present approach that should be retained? What criteria shall be used to select the newer mathematical experiences? How can some of the newer content be integrated into the existing curriculum? How can teachers become familiar with the newer mathematics? These are representative of the many questions and problems which must be solved before a new curriculum can be planned, written, and used effectively by the classroom teacher.

A variety of curricular approaches are being proposed and used. The City Schools of San Diego chose a practical approach which retains the values of the current program, but encompasses certain newer ideas and materials and provides an orderly transition from the present practice and content to a new and modified arithmetic program.

The development of a new arithmetic program received its initial start through action of the general elementary curriculum planning committees and the selection of primary and intermediate arithmetic subcommittees. These committees consisted of representative teachers, supervisors, and principals. Richard Madden of San Diego State College served as the general consultant and gave invaluable assistance in the formulation of the new curriculum.

The committee's initial assignment was to (1) study the newer trends and content in elementary mathematics; (2) develop plans for an improved program; (3) make specific suggestions to guide curriculum writers in preparing an arithmetic section for the grade-level curriculum guides. This was accomplished during three semesters of rather intensive work.

The next step was the appointment of two curriculum writers, one primary and one upper-grade. Elementary principals, working on an hourly basis during the school year, wrote the material and presented it to the subcommittee for evaluation and suggestions.

During the summer the materials were reviewed by an editing committee. By fall the new guide sections were printed in tentative form for trial use and evaluation by teachers in ten selected schools.

It was assumed that many modifications and changes would need to be made as teachers used the new guide sections. Therefore, the materials were placed in loose-leaf binders to facilitate modification.

The introductory information written to the teacher again emphasized the many changes which characterize the teaching of arithmetic today.

"To develop an arithmetic curriculum during this period of rapid change poses

many difficulties. Terminology is being revised, teaching techniques are changing and arithmetic books and tests are being revised and rewritten. Many of the ideas contained in this guide are not included at this time in the present State text series, but will probably appear in subsequent adoptions.

"Under these circumstances it seems logical to provide a brief, tentative guide for arithmetic which can be readily changed to meet the new developments in the field of arithmetic. It is hoped the material contained in this section will open some new vistas to teachers in the teaching of arithmetic and give practical assistance in the more effective utilization of State text materials. As rapidly as additional materials become available, as soon as research and experimentation have proved the value of new approaches, San Diego's arithmetic curriculum will be modified."

The new curriculum guides are organized on a grade-level basis. The program is continuous from level to level, and a standard format is maintained from grade to grade and subject to subject in order to facilitate the use of the publication as a reference.

The arithmetic section of the grade-level guides has the same statement of philosophy or point of view in all guides, Grades 1 through 6. The guides are designed to complement and supplement, rather than duplicate, the curricular structure and suggestions for teachers found in the teacher's edition of the current State text. They contain suggestions for using the State text effectively, such as identifying major developmental lessons, and also for effective use of the teacher's edition.

The basic program has changed only moderately. The continued reliance upon the curricular framework contained in the State text series provides a broad, solid core of instruction with which teachers have had experience for some time and which all pupils need.

Most of the significant changes between the current program and the one outlined in the new guide relate to (1) suggestions for more refined treatment of individual differences with particular emphasis on an improved program for the more able in arithmetic, (2) an increased emphasis upon the mathematical aspects of elementary arithmetic, and (3) definition of some of the mathematical language which has only recently come into use in elementary-school arithmetic.

The program for the more able in arithmetic provides for both vertical acceleration and horizontal enrichment. Teachers of Grades 3 through 6, who have a more able group in arithmetic, are encouraged to move into the work of the next grade with this group, after the work for the current year has been mastered. At the same time, the guide contains suggestions for enrichment through the development of greater depth of understanding of the mathematical principles underlying elementary arithmetic and their application in society. These include working with nondecimal systems of notation, positive and negative numbers exemplified through use of a number line, understanding of reciprocals, rate, ratio, properties of equations, and simple formulae. The pacing plan for the more able group provides for approximately one year of acceleration by the end of sixth grade.

The section dealing with the mathematical aspects of arithmetic lists the major mathematical concepts which are developed by the pupil as he progresses through Grades 1–6. These include among others, the concept of magnitude, cardinal number and number set, decimal place-value system of notation, measures, concepts of form and position, and the properties of numbers as used in the basic operations (commutative, distributive, and associative).

While the guide is aimed at providing the teacher new to the district with a ready reference regarding our arithmetic program and with specific suggestions for organizing the class for instruction, grouping, sample lesson plans for developmental

lessons, and suggestions for using various arithmetic materials, it also has in it a section which will be challenging to teachers who are already secure in arithmetic. This section on teacher background provides a "top drawer" for the teacher who wants to stand on tip-toe and reach, for the teacher who really wants to understand and develop the mathematical principles underlying elementary arithmetic.

While retaining a core of content and methodology which teachers understand and can teach effectively, the guide offers stimulation to acquire the vocabulary and the understanding which we hope will enable them to make the transition to the "new mathematics" without excessive confusion.

The committee which planned and directed the development of the grade-level arithmetic guides recognized that the actual value of these curriculum materials depended upon the use teachers made of them. Thus plans were made for a two-phase in-service education program consisting of:

1 A general introduction of the arithmetic guides to familiarize teachers with them and help the teachers to use them effectively;
2 An introduction of some of the new mathematical content included in the guides with specific suggestions for meeting individual differences. Fifth- and sixth-grade teachers had the greatest immediate need for this information.

The general introduction of the guides was done in individual school faculty meetings by principals, assisted by teachers. In preparation for these meetings, principals observed a demonstration faculty meeting presented by members of the arithmetic committee. The contents of the guides were carefully reviewed and specific sections were studied in detail, for example, Teacher Background, Enrichment, and Planning for Children of Varying Ability.

The second phase, introduction of new mathematical content, required more study and training on the part of leadership personnel. For this reason, organization of phase two was done centrally for the entire district. Three teams were organized, each consisting of a principal, chairman, and four team members. Each team served approximately 35 elementary schools.

Two training meetings were conducted by members of the arithmetic committee to familiarize team members with their role in the in-service training program. Then each team scheduled four meetings for the fifth- and sixth-grade teachers in its region of the city.

At the in-service training meetings for fifth- and sixth-grade teachers, team members offered specific suggestions for organizing to meet individual differences and introduced the following topics: the use of the equation in problem solving; rate, ratio, per cent; reciprocals and their use; use of the number line. Teaching materials were distributed and explained. Teachers were encouraged to study and use the materials provided and compile questions for future study groups.

The arithmetic guides have been well received. There has been noticeable growth in the effort made to meet individual differences, particularly the needs of able children. Widespread interest has been stimulated in new developments in mathematics. It is felt that the grade-level arithmetic guides are serving their purpose during this transition period in the development of the arithmetic curriculum. They have provided needed guidance, alerted teachers to new developments, and served to help teachers to develop readiness for continuing change.

# Don't count your chickens before they hatch

J. WILLIAM LAMBERT

*Dearborn Township District #7, Dearborn, Michigan*

*Mr. Lambert, formerly a mathematics and science teacher at Annapolis Junior High School in Dearborn Township District #7, is now assistant principal of the Mayfair-Polk Elementary School.*

In the first grade we feel that the arithmetic period should be a learning laboratory. The children should be involved in various concrete and meaningful experiences in which numbers play a vital role. These experiences may include grouping and manipulating objects, measuring and constructing objects, or dramatizing number experiences, all of which help the children acquire an understanding of number concepts.

Mrs. Sally Terry, a first-grade teacher at Mayfair-Polk Elementary School in Dearborn Township School District #7, has been providing a wonderful learning experience in arithmetic for her students. This experience, involving the hatching of chicken eggs, was offered to her children during the latter part of March and the first part of May, giving them an opportunity to apply many of the number facts that they had learned during the school year.

Before the fertilized chicken eggs were purchased, Mrs. Terry discussed with her children many facts concerning the hatching of eggs. The children found that they needed a wooden box about 15″×15″ to hold the eggs. They needed a container for water in the bottom of the box. Some children in the class had seen incubators used on the farm and suggested that light bulbs could be used as is "done on the farm" to provide the necessary heat. They investigated further to find that the temperature must be around 97° F. with a variance of no more than 95° to 100° F.

Mrs. Terry first introduced the use of the thermometer. The children had had experience in counting by two's, but to relate this to use with the thermometer was a little more difficult. A large thermometer was used to illustrate that each line on the thermometer represented two degrees. The children were all given many opportunities to use a thermometer and read the temperature. When she felt that most of the children were proficient in the elementary use of the thermometer, she proceeded, with the children's help, to record temperatures of the incubator on the chalkboard. This was done for several days before the eggs were placed in the incubator.

The thermometer and water were placed in the incubator. They had to experiment with different wattages of light bulbs to find one that produced heat that ranged between 95° and 100° F. They first tried a one-hundred-watt bulb. After a sufficient length of time the children checked the temperature inside the incubator and found that it was nearly 200° F. Finally, they agreed that a fifteen-watt light bulb was sufficient for their purposes for the temperature stayed between 95° and 100° F. for several days. Through this experiment the children were learning that a higher wattage bulb produces a greater reading on the thermometer.

The class found that a period of three weeks was necessary for the eggs to hatch. Before their project began, they had to have an understanding of how long a week is in relationship to number of days. They found by studying a calendar that a week represents seven days. They found that three groups of seven days in a week represented twenty-one days. In this phase of the experiment they were discovering the use of the calendar and the number relationships that are involved.[1]

The concept of telling time is also involved in this experiment. Children have the knowledge that clocks and watches tell the time at which specific things are to happen. Many children by the time they reach the first grade are able to tell by the position of the hands of the clock when it is time for certain television shows or when it is bedtime. As the clock is studied in the primary grades, the children learn that the long hand tells the number of minutes as it makes one complete revolution per hour and that the short hand only moves from one number to the next each time the long hand makes one complete revolution.[2] Mrs. Terry first developed the understanding of telling time by the hour and then developed the understanding of telling time by the hour and half-hour.

When the children became proficient in telling time, reading a thermometer, and the understanding of the relationships of weeks and days, Mrs. Terry purchased two dozen fertilized chicken eggs. A dozen is usually associated with the number of eggs in a carton. The class counted the dozen eggs as they placed them in the carton. In this way they were developing the idea that a dozen of anything represents twelve objects. They found that if one dozen eggs represented twelve eggs then two dozen eggs represented twenty-four. They did this by grouping the two dozen eggs. They also used the eggs for developing concepts of grouping such as two tens and four ones to make twenty-four.[3]

When the children were able to understand the above-mentioned concepts and were proficient in their use, the actual hatching of the chicken eggs was started. The eggs were placed in the incubator on April 25. A schedule was set up so that the children in groups of two could check the temperature in the incubator every half-hour that school was in session. All of the children were involved in this phase of the experiment. It was the children's responsibility to know when it was their turn and time to check. In this way they were using their understanding of telling time and also reading the thermometer. The children then had their readings placed on the chalkboard so that they had a daily record of the day's events. The eggs had to be turned over at the same time each day. Again, certain children were responsible for this. It was difficult for the children to know which eggs they had turned over so they placed an "X" on each egg. When they turned over the eggs, all the "X's" were showing one day; the following day no "X's" were showing. In this way the children were certain that all eggs had been turned each day.

On the third day two eggs were broken open so that the children could observe the chicken embryo's heartbeat. They found that the embryo remained alive outside of the shell for six hours after it had been opened. Again, the understanding of telling time and determining the length of time was involved in this experiment. The children realized that they no longer had two dozen eggs left. By counting the remaining eggs and by using the concept of "take away," they observed that twenty-two eggs remained.

The children recorded at the end of each day the number of days that the eggs had

[1] George E. Hollister and Agnes G. Gunderson, *Teaching Arithmetic in Grades I and II* (New York: D. C. Heath & Co., 1954), p. 113.

[2] *Ibid.*, p. 115.

[3] *Ibid.*, p. 84.

been in the incubator and the number of days remaining before the eggs were to hatch. The children were indirectly being exposed to the ideas of addition and subtraction.

The children had planned on opening other eggs at different intervals of the incubation period, but they decided that if more eggs were opened, less chickens would hatch. They also realized that if more were opened, there would not be a baby chick for each of them. At this point the one-to-one relationship was being exhibited by the children in the classroom.[4]

Unfortunately, five days before the eggs were to hatch the light bulb went out during the night. The children were first to notice this happening. The children checked the temperature of the incubator that morning and found that it had dropped to 70° F. We had no idea of how long the bulb had been out. We opened five more eggs before we observed body movement of the embryo chicken so we replaced the light bulb in hopes that some of the eggs would still hatch. In the children's arithmetic lesson that day they found that they had only seventeen eggs remaining. It was difficult for the children to realize that unfortunate things like this can happen, but Mrs. Terry handled the situation very well and the children's eagerness was again renewed for the eggs to hatch. Mrs. Terry discussed with the children that there was a possibility that none of the eggs would hatch due to the drop of the temperature.

Finally on May 16, the children observed that some holes had been pecked in the shells of the eggs. They could hear some peeping inside of the shells. The children were amazed and excited to observe four chicks emerge from their shells —wet and peeping!

I feel that Mrs. Terry did a wonderful job of exposing these children to the use of some of the basic arithmetic understandings through such a real and vivid experience. This project certainly illustrates that an arithmetic class need not be an entity in itself, but may be incorporated with science, social studies and everyday living.

[4] *Ibid.*, p. 66.

### Bibliography


Brueckner, Leo J., and Merton, Elda L. *Ready for Numbers.* Philadelphia: The John C. Winston Co., 1955.

Hollister, George E., and Gunderson, Agnes G. *Teaching Arithmetic in Grades I and II.* New York: D. C. Heath & Co., 1954.

Spitzer, Herbert F. *Practical Classroom Procedures for Enriching Arithmetic.* St. Louis: Webster Publishing Co., 1956.



Credit is also due to Mrs. Sally Terry, a first-grade teacher at the Mayfair-Polk Elementary School in Dearborn Township School District #7, Michigan.


"Research has been initiated to improve teacher training through television, to develop self-learning devices, and to prepare and test new materials in language, science, mathematics, history, and engineering."—From *Schools in Our Democracy*, Office of Education, U.S. Department of Health, Education, and Welfare.

# Planning an arithmetic fair

DOROTHY F. GILMAN, PRINCIPAL,
AND MEMBERS OF THE FACULTY
*Wolf Swamp Road School, Longmeadow, Massachusetts*

Each spring we have an exhibit to interpret to the parents of children in our school a particular subject-matter area from kindergarten through the sixth grade. In the past three years there have been a Science Fair, a Book Fair, and a Health Night at Wolf Swamp School.

Last September the idea of having an arithmetic fair as our next project was enthusiastically received by the teachers. They were anxious to stimulate continued improvement of the arithmetic program. We discussed our interpretation of this type of fair and our reasons for having such an event. We agreed that we wished to develop critical thinking concerning arithmetic concepts, to demonstrate grade continuity of the arithmetic programs, to foster creative use of arithmetic ideas and processes, and to encourage the developmental production of actual teaching and learning devices, models, exhibits, and games, through child-parent co-operation.

What major areas were we to cover? We divided the fair into eight categories having a specified section in our halls for each of the following categories: (1) measurement, (2) fractions, (3) addition and subtraction, (4) multiplication and division, (5) understanding the number system, (6) the language of arithmetic, (7) time, and (8) problem solving. Each faculty member selected an area for which he was to assume responsibility. This responsibility included collecting the entries, decorating the section, arranging the displays, and working with the children who would demonstrate the devices, games, and models for the night of the fair.

There were two types of awards—blue ribbons and Wolf Swamp Road Arithmetic Fair Emblems. The blue ribbons were awarded for the most outstanding work based on originality, neatness, practicability, thought, and effort. If a project was not considered a blue ribbon winner, but the child had put effort and thought into his creation of a device, game, or model, he was to be given an emblem. It was hoped that the true award for the child was the amount of learning that came from this creative activity and the fact that he could help to give more meaning to some phase of arithmetic for his class and other classes.

March became an exciting month. Children quickly became quite secretive over plans which led to much speculation and discussion. As the first arithmetic articles were brought in two weeks before the fair, children, who up to this point had not expended effort, became interested and decided to participate. Many children got another idea after bringing in the first exhibit and made a second or third device or game. When final count was taken, there were more entries than we had ever dreamed possible.

Anticipation heightened as arrangements for the exhibit began in the halls. Walls, tables, and display boards were used. The children were very impressed. This was really going to be something, and they made it very clear to their parents that this was an Arithmetic Fair that their parents could not afford to miss.

Since entries were brought in by children from kindergarten through the sixth

TRIP THROUGH
ARITHMETIC

Measures of length, liquid, weight
Changing measures from one unit to another

grades, there was a great variety of projects for each category and a wide range in the difficulty of the construction. There were on display electric boards, teaching machines, number charts, time-zone traveler computer, zoo of twos, flannel boards, countdown for rocket blast-off, counting scrapbooks, calendars, Chinese abacus, several types of counting frames, early thermometers, weight balancers, puzzles, scales, fraction wheels, fracoscope, fraction tree, flexible rulers, fraction boards, fraction pies, astronomy booklets, games, slide rulers, farm scenes to show grouping of objects, mural, dioramas, moving picture boxes to illustrate the history of different phases of arithmetic, boat- and birdhouses (with plans drawn to actual size of scale), and a hand-made dress (made according to measurements). The time category was well represented by water clocks, standing sundials, hourglasses, cuckoo clock, victrola clock, grandfather clock, and many other models of unusual description. There were just too many entries to enumerate or describe in this article. It was evident in many cases that arithmetic had a close relationship to other subject areas—social studies, science, hygiene, and art.

Parents often are asking for suggestions for giving children help at home. A paper was run off on the ditto machine entitled, "How Parents Can Help With Arithmetic At Home." This was given to the parents at the door as they entered for the fair. Since we wanted parents to have some fun at this big event, we set up a participation booth and had the following type of questions on our Participation Contest Paper: estimation, number puzzles, missing numbers, optical illusions, and guessing how many people were at the fair. For our winners, we had commercial games and devices as prizes.

For the final part of our program, we met in the assembly hall to hear a talk by Dr. Mark Atkinson on the subject of arithmetic in today's curriculum. Questions about arithmetic, submitted by parents and teachers, had been sent to Dr. Atkinson. As our guests sat waiting for the speaker to arrive, they were given a cross-number puzzle to work. Everyone had fun trying to figure out the answers.

From comments by parents and other visitors heard at the conclusion of our Arithmetic Fair, we felt that arithmetic is best learned when it has meaning and when it is realized that arithmetic can be fun.

---

## Horizons unlimited

**"We live in a world that can be made to blow up or grow up; a world having a race between high explosives and high intelligence; a world where scientific developments are many years ahead of our ability to understand the social, economic, moral, or spiritual implication. . . . The individual is our greatest asset."—Excerpts from luncheon address given by RAY O. MERTES at NCTM twenty-first summer meeting in Toronto**

# Just plain drill

JOSEPHINE K. COLEMAN *Chevy Chase, Maryland*

*Mrs. Coleman is a fourth-grade teacher at Rollingwood School in Chevy Chase, Maryland.*

Learning by repetition, in the early or later grades, has perhaps an undeservedly bad reputation that is based purely on *memoritor* work, lacking in motivation. The sorry but realistic allusions which have enriched our literature in the past two or three centuries from the pens of Lamb, Dickens, Captain Marryat, and Eggleston point up a succession of dreary, stolid, unimaginative educational experiences.

That these experiences were by no means unprofitable is attested by the literary skill which, after some fashion, they seemed to have engendered. The late H. L. Mencken was not alone in observing an obvious relationship between the business end of a hickory stick and learning. However, while one may admire Mr. Mencken's scholarship, or chuckle with appropriate reservations over his picturesque animadversions on educationists, the fact remains that the student finally produces best because of a self-interest that is not based on fear of punishment. Fortunately, in modern education, the attainment of skills has many motivational awards, aside from the negative.

Obviously, the success of so-called extracurricular things that students do because of a desire for status or self-expression of one sort or other—music, the dance, athletics, dramatics—depends on practice which is plain repetition—just plain drill! Hard work such as this generally requires some real approbation.

The satisfaction of doing something well, however, comes not always from the acclamation of the crowd, but from the recognition of self-improvement. Golf is a classic example wherein the player, in his soul, enjoys his improvement only by mathematical evidence. Bowling is another. So are shooting and billiards.

The competitor is improving himself through practice, without regard to outside influence, because nobody will help or hinder him; but he is stimulated by his own progress. Jumping, the javelin throw, and the shot-put are familiar examples.

Wilbur Wright once commented that Otto Lilienthal, the pioneer glider, and the first really successful man in his field, lost his life because he had not mastered the fundamental of practice. The glider had stayed aloft almost five hours, but he had worked at it for five years.

"Not even Methuselah," opined Wilbur, "would ever learn something like shorthand at the rate of one hour a year."

Can this line of reasoning be applied to the development of mathematical achievement and skills in the fourth grade? Our experience demonstrates that it can, and that such achievement is useful, important, eminently practical, and entirely manipulatory as a measure of student desire to improve.

We have used a project in our fourth-grade class in arithmetic that is designed to measure achievement graphically, to provide an incentive for improvement, and to give a student a basis for self-evaluation.

Our fourth graders had been taught the meaning and the mechanics of multiplication, but the time had arrived when we felt that the multiplication facts of the twelve tables should be remembered as readily as the basic subtraction and addition facts. Accordingly, for two weeks we gave a daily test of twenty-five problems. Each student recorded his progress on a

simple line graph. Interest heightened. Every student wanted to improve his own score. He didn't think of his neighbor's score. The drill was of so much interest, because of the graph-recording that we did not dare miss the daily drill.

Testing periods lasted for twenty minutes. The problems were dictated orally and were answered in writing. Papers were exchanged, with fifteen girls and fifteen boys checking. The value of this type of drilling and graph-recording was reflected to a degree in student response.

This went on for two weeks, and at the close of this period each fourth grader wrote an evaluation of his own progress and knowledge of multiplication. Then each averaged his own score.

"Now I can answer quicker than a wink," one child remarked, "when someone asks me what is seven times nine, or anything in multiplication."

"I started out," one child said, "below the seventy line, and didn't like that, so I decided to study and remember all the multiplication facts. My line graph shot up to the nineties, and the last three are on the hundred line. And now I have no trouble with short division."

Every teacher recognizes the desirability of having the facts at the student's finger tips. Drill is essential, but in this case the class found it interesting. We found that the practical application of a simple line graph was the motivating force which sparked the desire to do better. The self-improvement was remarkable and the children were happy while learning.

It was just plain drill, but the youngsters did not think of it as such!

---

## National Science Foundation Summer Fellowships

We wish to make known to our readers, particularly those teaching seventh- and eighth-grade mathematics, that the National Science Foundation plans to award on March 15, 1962, several hundred Summer Fellowships for Secondary School Teachers of Science and Mathematics. Those who receive fellowships must pursue a program of graduate-level work in the subject matter of science or mathematics. The fellowships will be awarded to support individually planned programs of study in the mathematical, physical, and biological sciences at a level that is acceptable by the fellowship institution toward an advanced degree in any of these subject-matter disciplines.

*The closing date for receipt of applications is January 2, 1962.* Information and application forms will be sent upon request addressed to Secondary School Fellowships, American Association for the Advancement of Science, 1515 Massachusetts Avenue, N.W., Washington 5, D.C.

A teacher may apply for fellowship support for one, two, or three summers. Each applicant chooses the college or university at which he wishes to study—there is no list of colleges and universities selected for this purpose. The fellowship program is completely separate from the NSF summer institute program. It is recommended that a program of study for the period of the fellowship be developed with the assistance of a staff member at the proposed fellowship institution. The program of study is a very important part of the application. Although some colleges and universities will not have announced their course offerings for the summer of 1962 by the deadline date for applications (January 2), the proposed program of study in the application should list specific courses from which the courses for study in the summer of 1962 will be chosen.

Prior teaching experience in a secondary school is required of all applicants, and no exceptions will be made to the requirements outlined in the announcement. Those without the requisite teaching experience and those who must complete undergraduate prerequisites before undertaking a program of graduate-level work should not apply for a fellowship under this program.

# The use of overlay charts

## A device for teaching some number concepts

RUTH MACHLIN, *Oakridge School, Arlington, Virginia*

The teaching device described is based on a hundred chart and a series of overlays. I used a portable flannel board and inserted several hooks on top from which the overlay papers were suspended. The paper used was 24″ by 18″ construction paper. A different color was used for each overlay to make the project attractive to the children. The children were second-graders with an excellent understanding of arithmetic. Counting in multiples, odd and even numbers, and bases other than base 10 were the understandings which were explored with this teaching aid.

In counting by 2's the hundred chart was at first displayed, uncovered (Fig. 1).

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Hundred Chart |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | |

*Figure* 1

It was then covered by the overlay which blocked out all the numerals not involved and left only this (Fig. 2):

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | | 4 | | 6 | | 8 | | 10 | 2's |
| | 12 | | 14 | | 16 | | 18 | | 20 | |
| | 22 | | 24 | | 26 | | 28 | | 30 | |
| | 32 | | 34 | | 36 | | 38 | | 40 | |
| | 42 | | 44 | | 46 | | 48 | | 50 | |
| | 52 | | 54 | | 56 | | 58 | | 60 | |
| | 62 | | 64 | | 66 | | 68 | | 70 | |
| | 72 | | 74 | | 76 | | 78 | | 80 | |
| | 82 | | 84 | | 86 | | 88 | | 90 | |
| | 92 | | 94 | | 96 | | 98 | | 100 | |

*Figure* 2

The same principle is used in counting by 3's, 4's, and 5's. Only the numerals in the desired series are left uncovered (Fig. 3, 4, and 5).

This, for bright youngsters, was really an adventure in numbers. It led to use of the term "divided by," to animated thought on the patterns which evolved, and to questions, such as "what would happen if we went over one hundred?"

To teach the concept of odds and evens, a chart labeled "odds" was laid over the hundred chart (Fig. 6), and the 2's chart was put to work to demonstrate "evens."

We found that numbers could be fascinating when we put aside thinking only in terms of base 10 and learned to work with different bases. In base 5, for example, the number 12 means one five and two ones, or the equivalent of 7 in base 10

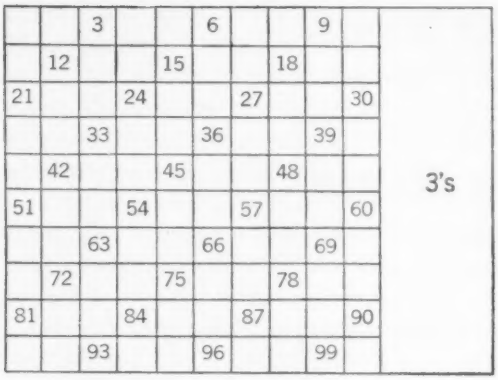



*Figure* 3

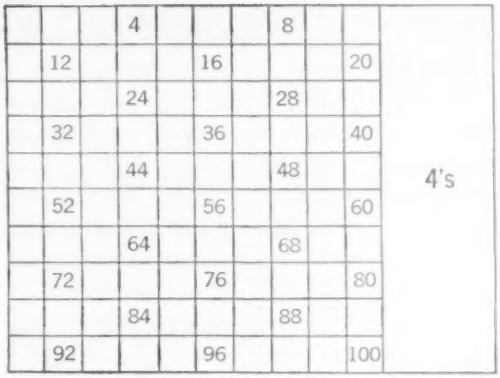



*Figure* 4

(Fig. 7). Base 6 extends our thinking, and 20 means two sixes and no ones, or 12 in base 10 (Fig. 8). Base 7 shows more numerals, and 20 here means two sevens and no ones, or 14 in base 10 (Fig. 9).

Since these were advanced second-graders, it was fun to see what would happen if more than one overlay was used at the same time. Therefore, when the 3's chart is put over the 2's chart, we found only the numbers which we use when counting by both 2's and 3's. Many are left out. Some children recognized that these numbers were also used in counting by 6's. (See Fig. 10.)

To go a little further, the 4's chart is superimposed on the 2's and 3's and only 8 numerals remain. We stop on each of these numbers when we count by 2's, by 3's, and by 4's (Fig. 11).

What will happen if we put our 5's chart over all the others? Which numerals will remain that we find in counting by 2's, 3's, 4's, and 5's? The answer, of course, is that only one little square will be left uncovered—60 (Fig. 12).

Only some of the children could go so far as to recognize 66 in base 7 as being 48 in base 10, but all were able to learn to count in these different bases and to realize that what we call 10's place in base 10 is actually bases' place rather than always 10's.

This device of a hundred chart and overlays was used at the end of the year for review purposes. In the coming year, I plan to use it as I teach the various ideas to which it lends itself. Whether we can go so far as different bases with the total class will depend on their understanding, but there is usually a group whose grasp of arithmetic is so strong that for them it can be used as enrichment. Last year, I used these charts in front of the room for dem-

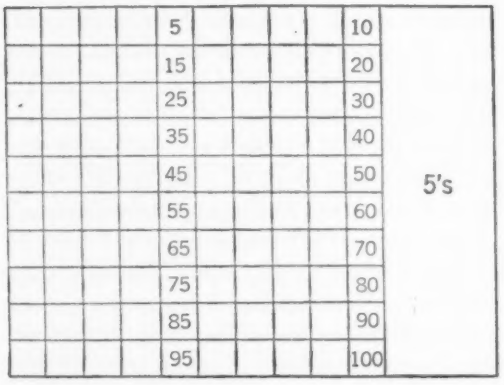



*Figure* 5

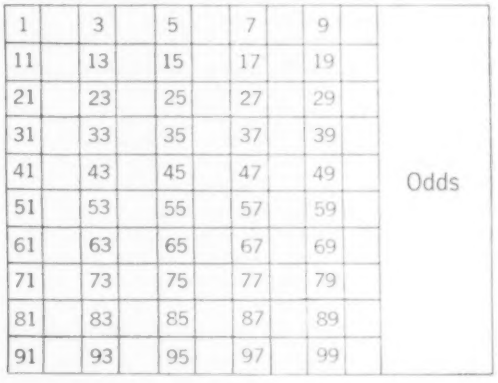



*Figure* 6

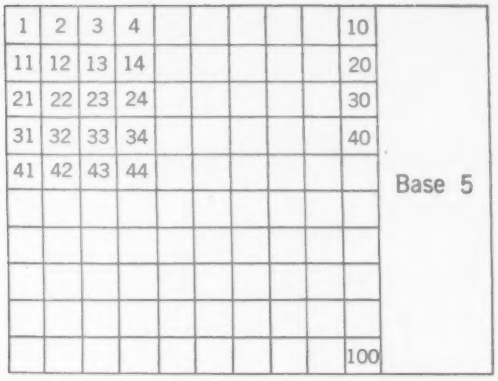



*Figure* 7

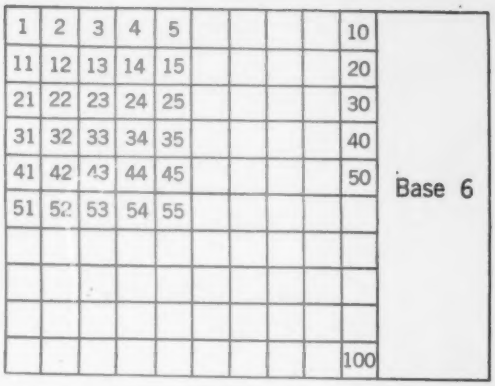



*Figure* 8

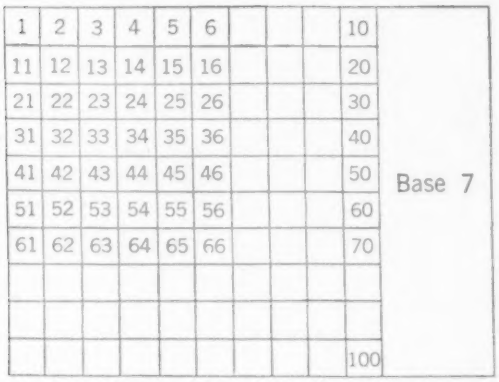



*Figure* 9

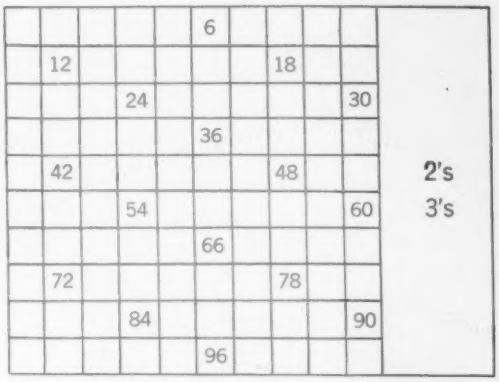



*Figure* 10

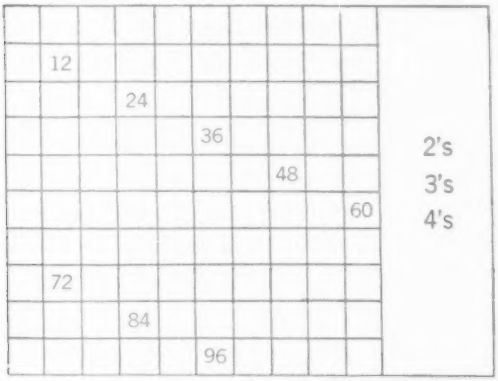



*Figure* 11

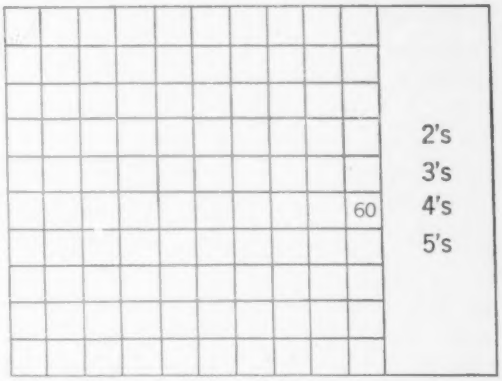



*Figure* 12

onstration purposes. Individual children came up and pointed out the various things we discussed. If made out of sturdier material, the series of overlays could be left in a corner of the room for children to handle as the need arises. To summarize, I found these charts useful as an additional tool to supplement other teaching aids, thought provoking, and attractive to the children.

*Editorial note:* Teachers of upper-elementary grade levels will see the implications of this device for helping children learn multiplication and division facts and for finding the common multiples for two or more numbers.

# The School Mathematics Study Group Project on Elementary-School Mathematics: A Progress Report

J. FRED WEAVER
*Boston University, Boston, Massachusetts*

An account of the School Mathematics Study Group's Project on Elementary-School Mathematics appeared in the January, 1961, issue of THE ARITHMETIC TEACHER. Now it is appropriate to review that account and bring it up to date.

### Origin and organization of the project

The following statement by Professor E. G. Begle, Director of the School Mathematics Study Group, provides helpful background for this report of the project's origin, organization, and present status:

"It is the objective of the School Mathematics Study Group to bring about an improvement in the teaching of mathematics in our schools. The increasing dependence of our society on science and technology makes this improvement imperative. The number of our citizens skilled in mathematics must be greatly increased and an understanding of the role of mathematics in our society is now a prerequisite for intelligent citizenship.

"In trying to bring about an improvement in school mathematics, it might seem at first glance that it would be best to start at the beginning, kindergarten, and to work up. However, our procedure has been just the opposite of this. We have set a goal to aim at, a decision as to what we would like to have our students know when they finish high school. For example, our goal for the college-capable student is a program in high school which will enable him to take in his first college year a substantial course in calculus. For the less capable student, our goal is a mastery and an understanding of as many mathematical skills as the student is capable of, and at the same time an understanding of the nature of mathematics and of the role of mathematics in our society.

"Having set our goals, we then work backwards. One of the School Mathematics Study Group projects which is currently under way is the design of courses for Grades 9 through 12. These will not only provide the mathematical prerequisites for a formal course in calculus but also as much informal and intuitive background for such a course as possible.

"Another of our projects is concerned with Grades 7 and 8. Here we are attempting to recast the program for these two years so that they will provide a much better intuitive background for the work of the higher grades.

"With the work on the curriculum for Grades 7 through 12 well under way, it became appropriate to consider the mathematics curriculum in the elementary school. The high-school mathematics curriculum rests on the foundation built in the first six grades, and an improvement in

these grades might allow a substantial strengthening of the program for the higher grades.

"For this reason, a conference on elementary-school mathematics was held at the Conrad Hilton Hotel in Chicago on February 13 and 14, 1959. The [54] participants included college and university mathematicians, high-school teachers, educational experts with special interest in arithmetic, supervisors, elementary-school teachers, psychologists, and representatives of scientific and government organizations having an interest in mathematics."[1]

The suggestions and recommendations growing out of this conference were referred to an ad hoc committee and translated into specific action proposals. In due time, the SMSG policy-making Advisory Committee approved the recommended downward extension of the Study Group's over-all work to include the elementary-school level, the National Science Foundation appropriated funds necessary to implement this new phase of SMSG activity, and a Panel on Elementary-School Mathematics was appointed to initiate and supervise the work of the new project. The members of this panel[2] met for the first time on January 30, 1960—within a year after the conference on elementary-school mathematics gave impetus to the extension of SMSG activity to embrace Grades K–6.

### National Council co-operation

Readers of THE ARITHMETIC TEACHER will be interested to learn of the facilitating role played by the National Council of Teachers of Mathematics in establishing the SMSG Project on Elementary-School Mathematics. In December, 1958, the Council's Elementary-School Curriculum Committee (ESCC) submitted a report to the NCTM Board of Directors in which a preliminary plan of action was outlined for a comprehensive study of the elementary-school mathematics curriculum. This report included the recommendation that the proposed study be tied in as directly as possible with an appropriate ongoing mathematics curriculum project at the secondary-school level.

Action soon followed. The Board approved the ESCC report and authorized the president of the Council and the chairman of ESCC to explore with the director of the School Mathematics Study Group the possibility of Council co-operation in organizing a downward extension of SMSG's work to include activities at the elementary-school level along the general lines proposed in the ESCC report. As a consequence of this fruitful exploration, the director of the School Mathematics Study Group actively involved the authorized NCTM representatives in consultation at strategic times during the course of developments that culminated in the SMSG Project on Elementary School Mathematics.

### Putting the program together

At the first meeting of the panel, mentioned earlier in this account, various circumstances and reasons prompted the decision to center project activity at this time on the mathematics program for Grades 4, 5, and 6. In March, 1960, the members of the panel, augmented by five other persons, worked together uninterruptedly for a full week preparing a detailed outline of a suggested mathematics program for the intermediate grades (4–6) that would be in keeping with the broad objectives and specific ongoing projects of the School Mathematics Study Group.

For eight weeks during the summer of 1960, more than twenty persons worked together intensively as a writing team on the Stanford University campus to translate the above outline into appropriate instructional material for both pupils and teachers. Fully half of the members of the writing team were classroom teachers or supervisors—persons directly and intimately associated with elementary schools. The other members of the writing team included some of the persons who had

prepared the program outline for Grades 4–6, along with additional mathematicians and mathematics educators who have a special interest in the mathematics curriculum at the elementary-school level.

Two small classes of children—one of beginning fourth-graders, the other of beginning sixth-graders—met daily during most of the eight-week period for mathematics instruction with a competent elementary-school teacher. These classes permitted members of the writing team to have portions of their material tried out immediately as it was written, thus making it possible to settle some crucial questions on an empirical basis.

The instructional units which were developed reflected the mathematical spirit which permeates the whole of the SMSG effort at all levels. In particular, these three things were much in evidence: (1) an emphasis upon mathematical structure, (2) the inclusion of a substantial amount of geometric content, and (3) an extension of arithmetic content to embrace topics normally not introduced at the intermediate-grade level. (Each of these things is in evidence in the list of unit titles which appears later in this report.)

### Extensive trial of the program

Nineteen of the projected 25 units[3] were available for classroom trial during the 1960–61 school year. This use of the SMSG mathematics materials for Grades 4–6 was concentrated in eight experimental centers located from coast to coast. Each of these centers generally included twelve classrooms—six in Grade 4, three in Grade 5, and three in Grade 6. In keeping with SMSG policy, closely associated with each center was a college or university faculty member qualified to serve as a mathematics consultant to the elementary-school teachers who were using the experimental units with their classes. Similar consultant service was provided in the case of the 20 additional "points" which served to augment the work of the eight centers. All in all, the SMSG materials for Grades 4–6 were used in an exploratory way during the 1960–61 school year with approximately 4,000 children.

Teachers and mathematics consultants in the experimental centers submitted informative reports covering strengths and weaknesses revealed in the units on the basis of their use last year. These reports were invaluable to the writing team which worked for eight weeks on the Yale University campus during the summer of 1961, revising previously written units and preparing new units to round out the program for Grades 4–6. (With some exceptions, the 1961 writing team was composed of the same persons who served on the 1960 writing team.) The program as a whole has been strengthened in a variety of ways which include changes in approach to some mathematical ideas and rearrangement of the sequence of units to provide more effectively for the progressive development of mathematical concepts and skills.

### The 1961-62 program

The units which comprise the 1961–62 SMSG program for Grades 4–6 are listed below.

*Grade* 4

EA 101—Concept of Sets
EA 102—Numeration
EA 103—Properties and Techniques of Addition and Subtraction, I
EA 104—Properties of Multiplication and Division
EA 105—Sets of Points
EA 106—Properties and Techniques of Addition and Subtraction, II
EA 107—Techniques of Multiplication and Division
EA 108—Recognition of Common Geometric Figures
EA 109—Linear Measurement
EA 110—Concept of Fractional Number

*Grade* 5

EB 111—Extending Systems of Numeration

EB 112—Factors and Primes
EB 113—Extending Multiplication and Division
EB 114—Congruence of Geometric Figures
EB 115—Addition and Subtraction of Fractional Numbers
EB 116—Measurement of Angles
EB 117—Area
EB 118—Ratio
EB 119—(Supplementary Review Exercises)

*Grade 6*

EC 121—Exponents
EC 122—Multiplication of Fractional Numbers
EC 123—Side–Angle Relationships of Triangles
EC 124—The Integers
EC 125—Coordinates
EC 126—Division of Fractional Numbers
EC 127—Volume
EC 128—Organizing and Describing Data
EC 129—(Supplementary Review Exercises)
EC 130—Sets and Circles

During the current school year, 1961–62, these 29 units are being used in the same experimental centers that were operative last year. When possible the fifth-grade classes are composed of children who used the SMSG program in Grade 4, and the sixth-grade classes are composed of children who used available SMSG units in Grade 5. Again, reports from center teachers and consultants will provide useful information relative to the program in action.

In addition to the center classes, the SMSG materials are being used in a greater number of selected "points" during 1961–62 than was the case in 1960–61. Special combinations and sequences of units are provided for fifth- and sixth-grade classes which did not use the SMSG program last year. All in all, approximately 32,000 children in Grades 4–6 are using the 1961–62 SMSG materials.

At appropriate times in the future, further progress reports relating to the School Mathematics Study Group Project on Elementary-School Mathematics will appear in the SMSG *Newsletter*[4] and in this section of THE ARITHMETIC TEACHER.

**Notes**

1. Quoted from the SMSG *Report of a Conference on Elementary School Mathematics.*

2. The members of the SMSG Panel on Elementary-School Mathematics are: E. G. Begle (Stanford University), *ex officio*; E. Glenadine Gibb (State College of Iowa), W. T. Guy (University of Texas), S. B. Jackson (University of Maryland), Irene Sauble (Detroit Public Schools), M. H. Stone (University of Chicago), and J. F. Weaver (Boston University).

3. The 19 units used during 1960–61, and accompanying teacher commentaries, are available in bound form for teacher education and related uses. The complete set of *Mathematics for the Elementary School* 1960–61 units and commentaries may be purchased ($10 per set) from the School Mathematics Study Group, Box 2029, Yale Station, New Haven, Connecticut.

4. Interested persons wishing to receive future issues of the SMSG *Newsletter* may request to have their names placed on the mailing list by writing to Mr. George L. Roehr, Assistant to the Director, School Mathematics Study Group, School of Education, Stanford University, Stanford, California.

# Books and materials

*How To Use the Arithmetic You Know.* Geoffrey Mott-Smith. New York: Sterling Publishing Co. Cloth, 128 pp., $2.95.

This is the last book to have been written by the late Mr. Mott-Smith. It reflects his broad knowledge and appreciation of mathematics in all its forms and demonstrates again his ability to make a given subject meaningful.

A striking feature of this book is its broad coverage considering its size. It begins with a very brief discussion of our number system followed by a review of addition, subtraction, multiplication, and division. Checking by digital roots and short cuts may add to the reader's efficient use of these operations.

Other subjects dealt with are factoring, common and decimal fractions, ratio and proportion, powers and roots, logarithms, the slide rule, business arithmetic, measurement, scaling, graphs, and probabilities. There is nothing new or striking in this material other than the simple manner in which it is presented. However, as the title indicates, the reader is expected to have had some previous knowledge.

In practice, the book is a brief, fairly complete reference that would prove valuable to students and adults in solving specific problems or reviewing concepts. The use of this book could cause readers to seek more detailed information.

I doubt if many people would want to read through the entire book but there is something here for almost everyone.

PETER PIERRO
*Curriculum Co-ordinator*
*Plainfield, Illinois*

*Providing Ways for Children To Learn Mathematics in the Primary Grades,* A Curriculum Guide. Board of Education. Crowley, Louisiana: Acadia Parish Schools. Paper, 105 pp., $3.00.

A curriculum guide is a vital resource for the classroom teacher for at least three reasons: (1) it provides a suggested course of study; (2) it supplements the teacher's knowledge of materials and teaching procedures; and (3) it furnishes continuity to the instructional program in a particular school system. Nevertheless, many schools do not provide their teachers with carefully developed curriculum guides. One reason for the scarcity of good guides is the amount of time, money, and effort necessary for their preparation.

An excellent curriculum guide for mathematics in the primary grades has been developed recently in the Acadia Parish Schools, Crowley, Louisiana, through the co-operative efforts of 89 primary teachers, 18 principals, the mathematics supervisor, and the superintendent of schools. This school system is to be commended for including so many of its personnel on this project. The result is a guide which provides the teachers with a modern point of view regarding the teaching of mathematics in the primary grades, along with a comprehensive list of objectives for Grades 1, 2, and 3. Suggested procedures for carrying out the instructional program in each of the three grades include a variety of materials and procedures with emphasis upon meeting the individual needs of the children. Many pictures and illustrations of classroom activities provide teachers with a visual approach to developing

the basic concepts of mathematics. An outstanding list of teaching aids is arranged in the order of basic concepts to be developed. Both the price of each aid and its source increase the quality of this list. A complete list of both professional books for teachers and supplementary books for children, arranged in order of number concepts adds to the completeness of the guide as an outstanding resource for teachers.

Several questionable statements appear in the guide, such as: "*No attempt* should be made to add or subtract numbers until children understand the numbers involved," (p. 24); "The *secret* to easy mastery of addition and subtraction facts . . . " (p. 28); "What is $x$" (p. 33); "Two 2's *make* 4," etc. (p. 37); "The numbers seem to revolve around five." (p. 49); "concrete materials," "semi-concrete materials," and "abstract symbols" (p. 50 and p. 82); "How many 6's *are* 12," etc. (p. 56); and "$3\overline{)\$9.63}$ Read $\frac{1}{3}$ of $9.63," etc. (p. 69). In spite of such statements, the new curriculum guide of the Acadia Parish Schools is an excellent one and should fulfill its major objective of assisting teachers in providing ways for children to learn mathematics in the primary grades.

LONIE R. RUDD
*Tufts University*
*Medford, Massachusetts*

*Ruler, Compasses, and Fun.* Ali R. Amir-Moez. Lafayette, Indiana: Lafayette Printing Co., 1961. Paper, 31 pp., $0.65.

*Ruler, Compasses, and Fun* offers intermediate- and upper-grade teachers a guide for their pupils who would like to construct interesting and attractive designs with the use of a ruler and a pair of compasses.

Because of the large pages ($8\frac{1}{2}$" by 11"), the drawings are of greater size than usual. This, together with the large print, should enable the pupil to follow directions with ease.

The honeycomb and the Persian mosaic designs are explained in full detail, by first directing the pupil in the construction of the regular hexagon. The constructions of the regular pentagon and regular octagon lead to the designs of the five- and eight-pointed stars, respectively. Further directions include, among others, the construction of a line parallel to a given line, a line tangent to a circle, tangent circles, and a perpendicular to a line through a point not on the line.

Lamentably, some errors exist. On page 11 for the construction of a perpendicular to a line at a point, the direction " . . . choose two equal pieces on both sides of O" is not very meaningful, to say the least. On page 29 the reader finds that " . . . the lines which bisect $AB$ and $BC$ (two chords) cut each other at the center O of the circle," which, of course, is true only if each bisecting line is also perpendicular to a chord.

A more serious error, in the reviewer's opinion, is one of omission. The author has not provided any questions by which the pupil may discover for himself certain geometric relationships. By way of example, on page 16 the pupil is instructed to bisect the four right angles formed by two perpendicular diameters and to connect the resulting eight points on the circle to obtain a regular octagon. It would have been worthwhile to ask about the polygons formed if this process were to be continued. On page 30, with the directions for finding the perpendicular to a line through a point not on the line, the question could have been raised as to whether or not another such perpendicular different from the first might be found.

If a few errors are corrected and challenging questions are supplied, pupils should profit greatly from the use of *Ruler, Compasses, and Fun.* This book contains good supplementary material for informal geometry in fourth through ninth grade.

A. JERRY SHRYOCK
*Western Illinois University*
*Macomb, Illinois*

# THE ARITHMETIC TEACHER

*A journal of*

*The National Council of Teachers of Mathematics*

(*Incorporated*)

## Classified Index, Volume VIII

## 1961

---

E. Glenadine Gibb

*Editor*

*State College of Iowa, Cedar Falls, Iowa*

# Author Index—1961


ARANITI, FRANK C., The use of "1" in building concepts, Oct., 299–300

ASHLOCK, ROBERT B., The number line in the primary grades, Feb., 75–76

BENDER, MARVIN L., Dividing by zero, April, 176–179

BOTTS, TRUMAN, Numbers, sets, and counting, Oct., 281–286

BOYD, CLAUDE C., An in-service mathematics education program for intermediate-grade teachers, Feb., 65–68

BROWN, FRANCIS R., book reviews, Feb., 83; Mar., 138

BRUNE, IRVIN H., book review, Feb., 85

———, Geometry in the grades, May, 210–219

CHAMBERLIN, DON, San Diego's new arithmetic program, Dec., 422–424

CLARK, JOHN R., Looking ahead at instruction in arithmetic, Dec., 388–394

COLEMAN, JOSEPHINE K., Just plain drill, Dec., 431–432

CORMIER, ROMAE J., Subtraction by complement-addition-complement, Nov., 344

CULP, NELLIE, A Psalm of Lamentation or The Psalm of the Empty Set, Oct., 296

CUNNINGHAM, GEORGE S., More on divisibility by seven and thirteen, April, 180–181

DAMM, FRANCES L., book review, Feb., 84

DAVIS, CATHERINE LINN, Highlights of a summer conference, Jan., 14–18

DEANS, EDWINA, assistant editor, In the classroom, Jan., 27–31; Feb., 77–80; Mar., 131–134; April, 189–191; May, 251–254; Oct., 297–300; Nov., 368–372; Dec., 433–435

———, Arithmetic in science and social studies, April, 189–191

———, First graders use numbers in opening their school day, Nov., 368–372

———, Grouping—an aid in learning multiplication and division facts, Jan., 27–31

———, Independent work in arithmetic, Feb., 77–80

———, Remainders in division and a floor number line, Mar., 131–134

———, A third grade enjoys arithmetic, May, 251–254

DEVAULT, M. VERE, An in-service mathematics education program for intermediate-grade teachers, Feb., 65–68

DOGGETT, WILLIAM H., book review, Oct., 307–308

DRISCOLL, LUCY E., Ordered pairs, patterns, and graphs in fourth grade, Mar., 127–130

DUNCAN, ERNEST R., E pluribus unum—a brief discussion on the "law of one," Dec., 413–415

DUTTON, WILBUR H., University students' comprehension of arithmetical concepts, Feb., 60–64

FAULK, CHARLES J., An approach to problem-solving, April, 157–160

FISHER, ALAN A., The peg board—a useful aid in teaching mathematics, April, 186–188

FLAGG, ELINOR B., book review, Mar., 139

GIBB, E. GLENADINE, editor, About our editorial staff, Jan., 1–3

———, Are you planning to submit a manuscript? May, 241

———, As we read, May, 209

———, From the editor's desk—, Dec., 385–386

GILMAN, DOROTHY F., and MEMBERS OF THE FACULTY, Planning an arithmetic fair, Dec., 428–430

GUGGENHEIM, FRED, Classroom climate and the learning of mathematics, Nov., 363–367

GUNDLACH, B. H., The Greater Cleveland Mathematics Program, April, 192–195

HAGGERTY, JOHN B., Projects make mathematics more interesting, April, 172–175

HAMILTON, E. W., associate editor, As we read, Jan., 4; Feb., 49; Mar., 97; April, 145–146; Oct., 273, 286; Nov., 337–338; Dec., 386–387

———, Number systems, fad or foundation? May, 242–245

HAMMOND, ROBERT C., A device for practice with common denominators and addition of unlike fractions, Nov., 373
HARDGROVE, CLARENCE ETHEL, assistant editor, Reviews, books and materials, Jan., 36–38; Feb., 83–85; Mar., 138–140; April, 196; May, 261–264; Oct., 307–308; Nov., 377; Dec., 417, 440–441
HARRISON, K. L., Testing—without tests, Jan., 21
HAWLEY, NEWTON S., Geometry for primary grades, Nov., 374–376
HEARD, IDA MAE, Developing concepts of time and temperature, Mar., 124–126
HILAIRE, PAUL A., Let's take a look at division, May, 220–225
HLAVATY, JULIUS H., Will you contribute to a forthcoming yearbook? Jan., 20; Feb., 68
HOHN, FRANZ E., Teaching creativity in mathematics, Mar., 102–106
HOSIER, MAX, Some basic geometric ideas for the elementary teacher, Nov., 357–362
HOUSTON, W. ROBERT, An in-service mathematics education program for intermediate-grade teachers, Feb., 65–68
JACKSON, HUMPHREY C., Creative thinking and discovery, Mar., 107–111
JENNINGS, THOMAS J., Discovering bar graphs by degrees, Dec., 416–417
———, Tease them to think, May, 246–247
KAPRELIAN, GEORGE, Attitudes toward a television program—patterns in arithmetic, Dec., 408–412
KEFFER, EUGENE R., Individualizing arithmetic-teaching, May, 248–249
KLAS, WALTER L., Problems without numbers, Jan., 19–20
KOENKER, ROBERT H., Mental arithmetic, Oct., 295–296
KOLESNIK, THEODORE S., book review, Nov., 377
LAMBERT, J. WILLIAM, Don't count your chickens before they hatch, Dec., 425–427
LANDRY, THOMAS R., An approach to problem-solving, April, 157–160
LANSDOWN, BRENDA, Creating mathematicians, Mar., 98–101
LERCH, HAROLD H., Arithmetic instruction changes pupils' attitudes toward arithmetic, Mar., 117–119
———, Intra-class grouping for arithmetic instruction: critique and criteria, Dec., 404–407
MACHLIN, RUTH, The use of overlay charts, Dec., 433–435
MADAUS, GEORGE F., Evaluation of a mathematics program, Dec., 418–421
MASCHO, GEORGE, Familiarity with measurement, April, 164–167
MCKNIGHT, BLAIR A., Sets and numbers in grade one, 1959–60, Oct., 287–290
MCLAUGHLIN, JACK W., A new approach to an old problem, Mar., 112–116
MICHALOV, MARY, The versatile number runner, April, 182–185
MILLER, HERBERT F., book review, May, 261–262
MUELLER, FRANCIS J., On the fraction as a numeral, May, 234–238
NELSON, L. DOYAL, Mathematical competence of prospective elementary teachers in Canada and in the United States, April, 147–151
NULTON, LUCY, Arithmetic: arthritis or adventure? Nov., 345–349
OSBORN, ROGER, The use of models in the teaching of mathematics, Jan., 22–24
PACE, ANGELA, Understanding and the ability to solve problems, May, 226–233
PARSONS, CYNTHIA, Unusual arithmetic, Feb., 69–74
PECK, LYMAN C., book reviews, Jan., 36–38; Oct., 307
PETERSON, WAYNE, A case in point, Jan., 10–13
PIERRO, PETER, book review, Dec., 440
PINNEY, GRANT C., Grouping by arithmetic ability—an experiment in the teaching of arithmetic, Mar., 120–123
POWELL, LUCILE B., A spark of enthusiasm, May, 239–241
PRAY, RICHARD H., The metric system is simple! April, 179
RAPPAPORT, DAVID, Percentage—noun or adjective? Jan., 25–26
RECKZEH, JOHN K., E pluribus unum—a brief discussion on the "law of one," Dec., 413–415

Rivera, Emilio, Some new approaches to old arithmetic, Oct., 291–294
Rose, Alvin W., Intelligence, sibling position, and sociocultural background as factors in arithmetic performance, Feb., 50–56
Rose, Helen Cureton, Intelligence, sibling position, and sociocultural background as factors in arithmetic performance, Feb., 50–56
Rudd, Lonie E., book reviews, April, 196; May, 263–264; Dec., 440–441
———, Enrichment for the talented in arithmetic: a local program for grades 4, 5, and 6, Mar., 135–137
Rutland, Leon, Some basic geometric ideas for the elementary teacher, Nov., 357–362
Schaaf, William L., Mathematics as a cultural heritage, Jan., 5–9
Seeger, Raymond J., Fingerprints, Nov., 339–344
Self, William, San Diego's new arithmetic program, Dec., 422–424
Shine, Aileen, Relationship between arithmetic achievement and item performance on the Revised Stanford-Binet Scale, Feb., 57–59
Shryock, A. Jerry, book review, Dec., 441
Sparks, Jack N., Arithmetic understandings needed by elementary-school teachers, Dec., 395–403
Sueltz, Ben A., A time for decision, Oct., 274–280
Thorpe, Cleata B., Those problem-solving perplexities, April, 152–156
Suppes, Patrick, Sets and numbers in grade one, 1959–60, Oct., 287–290
Unkel, Esther R., Children are naturals at solving word problems, April, 161–163
———, Teaching multiplication facts need not be grim, Oct., 297–299
Weaver, J. Fred, assistant editor, Experimental projects and research, Jan., 32–35; Feb., 81–82; Mar., 135–137; April, 192–195; May, 255–260; Oct., 301–306; Nov., 374–376; Dec., 436–439
———, Analysis of research in the teaching of mathematics: 1957 and 1958, Feb., 81–82
———, The School Mathematics Study Group Project on Elementary-School Mathematics, Jan., 32–35; Dec., 436–439
———, Research on elementary-school mathematics—1960, May, 255–260; Oct., 301–306
Willerding, Margaret F., Other number systems—aids to understanding mathematics, Nov., 350–356
Wilson, Guy M., Why do pupils avoid mathematics in high school? April, 168–171
Wilson, J. Fred, San Diego's new arithmetic program, Dec., 422–424
Worth, Walter H., Mathematical competence of prospective elementary teachers in Canada and in the United States, April, 147–151
Zimmerman, Ronald, A method for checking addition, April, 181

## Title Index—1961

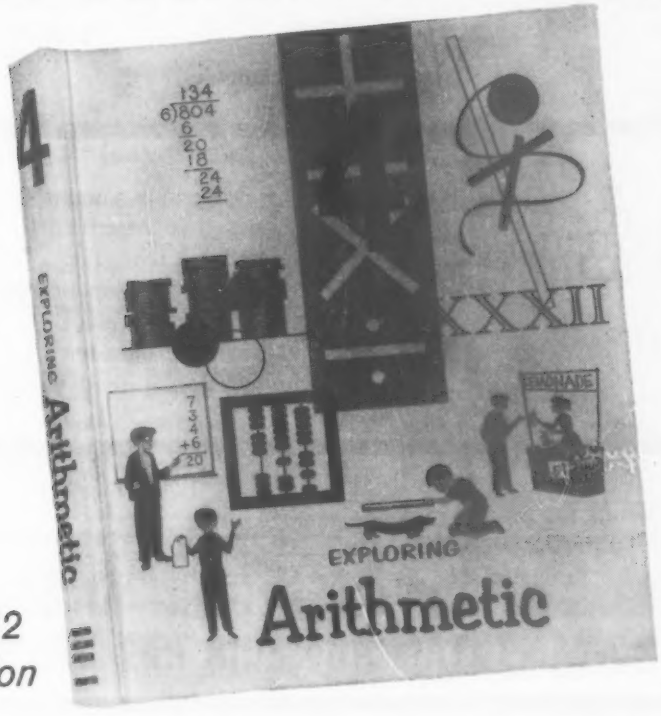
About our editorial staff, E. Glenadine Gibb, editor, Jan., 1–3
An approach to problem-solving, Charles J. Faulk and Thomas R. Landry, April, 157–160
Analysis of research in the teaching of mathematics: 1957 and 1958, J. Fred Weaver, assistant editor, Feb., 81–82
Are you planning to submit a manuscript? E. Glenadine Gibb, editor, May, 241
Arithmetic: arthritis or adventure? Lucy Nulton, Nov., 345–349
Arithmetic in science and social studies, Edwina Deans, assistant editor, April, 189–191
Arithmetic instruction changes pupils' attitudes toward arithmetic, Harold H. Lerch, Mar., 117–119

Arithmetic understandings needed by elementary-school teachers, JACK N. SPARKS, Dec., 395–403
As we read, E. GLENADINE GIBB, editor, May, 209
As we read, E. W. HAMILTON, associate editor, Jan., 4; Feb., 49; Mar., 97; April, 145–146; Oct., 273, 286; Nov., 337–338; Dec., 386–387
Attitudes toward a television program—patterns in arithmetic, GEORGE KAPRELIAN, Dec., 408–412
Book reviews
*Arithmetic for the Modern Age*, Aaron Bakst, reviewer—PECK, Oct., 307
*The Arithmetic Handbook*, Martin H. Ivener, reviewer—BROWN, Feb., 83
*Elementary and Junior High School Mathematics Library*, Clarence Ethel Hardgrove, reviewer—DAMM, Feb., 83
*Exploring Mathematics on Your Own*, Donovan A. Johnson, William H. Glenn, reviewer—MILLER, May, 261
*Extending Mathematical Ideas*, John L. Marks, James R. Smart, and Irene Sauble, reviewer—KOLESNIK, Nov., 377
*Geometry for Juniors*, Grace A. Moss, reviewer—BRUNE, Feb., 84
*How to Use Algebra in Everyday Life*, Editors of Sterling, reviewer—DOGGETT, Oct., 307
*How To Use the Arithmetic You Know*, Geoffrey Mott-Smith, reviewer—PIERRO, Dec., 440
*Magic House of Numbers*, Irving Adler, reviewer—BROWN, Mar., 138
*Numbers Old and New*, Irving and Ruth Adler, reviewer—BROWN, Mar., 138
*Providing Ways for Children To Learn Mathematics in the Primary Grades*, Board of Education, Crowley, La., reviewer—RUDD, Dec., 440–441
*Ruler, Compasses, and Fun*, Ali R. Amir-Moez, reviewer—SHRYOCK, Dec., 441
*Seeing Through Arithmetic Tests for Grades 3–6*, Maurice L. Hartung, Henry Van Engen, E. Glenadine Gibb, and Lois Knowles, reviewer—RUDD, May, 263
*The Story of Mathematics: Geometry for the Young Scientist*, Hy Ruchlis and Jack Englehardt, reviewer—RUDD, April, 196
*Using the Language of Algebra in Arithmetic*, William H. Glenn, William G. Mehl, Dean S. Rassmussen, reviewer—FLAGG, Mar., 138
*The World Book Encyclopedia*, J. Morris Jones (editor in chief), reviewer—PECK, Jan., 36
Books and teaching aids received, Jan., 38; Feb., 85; Mar., 140; May, 264; Oct., 308; Dec., 417
A case in point, WAYNE PETERSON, Jan., 10–13
Children are naturals at solving word problems, ESTHER R. UNKEL, April, 161–163
Classroom climate and the learning of mathematics, FRED GUGGENHEIM, Nov., 363–367
Creating mathematicians, BRENDA LANSDOWN, Mar., 98–101
Creative thinking and discovery, HUMPHREY C. JACKSON, Mar., 107–111
Developing concepts of time and temperature, IDA MAE HEARD, Mar., 124–126
A device for practice with common denominators and addition of unlike fractions, ROBERT C. HAMMOND, Nov., 373
Discovering bar graphs by degrees, THOMAS J. JENNINGS, Dec., 416–417
Dividing by zero, MARVIN L. BENDER, April, 176–179
Don't count your chickens before they hatch, J. WILLIAM LAMBERT, Dec., 425–427
Enrichment for the talented in arithmetic: a local program for grades 4, 5, and 6, LONIE E. RUDD, Mar., 135–137
E pluribus unum—a brief discussion on the "law of one," JOHN K. RECKZEH and ERNEST R. DUNCAN, Dec., 413–415
Evaluation of a mathematics program, GEORGE F. MADAUS, Dec., 418–421
Experimental projects and research, J. FRED WEAVER, assistant editor, Jan., 32–35; Feb., 81–82; Mar., 135–137; April, 192–195; May, 255–260; Oct., 301–306; Nov., 374–376; Dec., 436–439
Familiarity with measurement, GEORGE

Mascho, April, 164–167
Fingerprints, Raymond J. Seeger, Nov., 339–344
First graders use numbers in opening their school day, Edwina Deans, assistant editor, Nov., 368–372
On the fraction as a numeral, Francis J. Mueller, May, 234–238
From the editor's desk, E. Glenadine Gibb, editor, Dec., 385–386
Geometry for primary grades, Newton S. Hawley, Nov., 374–376
Geometry in the grades, Irvin H. Brune, May, 210–219
The Greater Cleveland Mathematics Program, B. H. Gundlach, April, 192–195
Grouping—an aid in learning multiplication and division facts, Edwina Deans, assistant editor, Jan., 27–31
Grouping by arithmetic ability—an experiment in the teaching of arithmetic, Grant C. Pinney, Mar., 120–123
Highlights of a summer conference, Catherine Linn Davis, Jan., 14–18
Independent work in arithmetic, Edwina Deans, assistant editor, Feb., 77–80
Individualizing arithmetic-teaching, Eugene R. Keffer, May, 248–249
An in-service mathematics education program for intermediate-grade teachers, W. Robert Houston, Claude C. Boyd, M. Vere DeVault, Feb., 65–68
Intelligence, sibling position, and sociocultural background as factors in arithmetic performance, Alvin W. Rose and Helen Cureton Rose, Feb., 50–56
In the classroom, Edwina Deans, assistant editor, Jan., 27–31; Feb., 77–80; Mar., 131–134; April, 189–191; May, 251–254; Oct., 297–298; Nov., 368–372; Dec., 433–435
Intra-class grouping for arithmetic instruction: critique and criteria, Harold H. Lerch, Dec., 404–407
Just plain drill, Josephine K. Coleman, Dec., 431–432
Let's take a look at division, Paul A. Hilaire, May, 220–225
Looking ahead at instruction in arithmetic, John R. Clark, Dec., 388–394
Mathematical competence of prospective elementary teachers in Canada and in the United States, L. Doyal Nelson and Walter H. Worth, April, 147–151
Mathematics as a cultural heritage, William L. Schaaf, Jan., 5–9
In memoriam, May, 264
Mental arithmetic, Robert H. Koenker, Oct., 295–296
A method for checking addition, Ronald Zimmerman, April, 181
The metric system is simple! Richard H. Pray, April, 179
More on divisibility by seven and thirteen, George S. Cunningham, April, 180–181
The National Council of Teachers of Mathematics, Annual financial report, Oct., 313
———, Articles of Incorporation, Oct., 315–316
———, The 1961 budget, Oct., 314–315
———, Bylaws, Oct., 316–318
———, Committees and Representatives, 1960–1961, Feb., 86–89
———, Committees and Representatives (1961–1962), Oct., 319–320
———, Convention previews, Feb., 89–90
———, Minutes of the Annual Business Meeting, Oct., 309–312
———, More about 1960–61 committees, April, 197
———, Newly elected officers, May, 238
———, Notice of Annual Business Meeting, Jan., 26; Feb., 76
———, Registrations at NCTM meetings, Nov., 378–380
———, Report of the Membership Committee, Oct., 321–325
———, Report of the Nominating Committee, Jan., 39–45
———, State representatives, 1960–61, May, 265–266
———, Twenty-First Summer Meeting, May, 267–268
A new approach to an old problem, Jack W. McLaughlin, Mar., 112–116
The number line in the primary grades, Robert B. Ashlock, Feb., 75–76
Numbers, sets, and counting, Truman Botts, Oct., 281–286
Number systems, fad or foundation?

E. W. Hamilton, May, 242–245
Ordered pairs, patterns, and graphs in fourth grade, Lucy E. Driscoll, Mar., 127–130
Other number systems—aids to understanding mathematics, Margaret F. Willerding, Nov., 350–356
The peg board—a useful aid in teaching mathematics, Alan A. Fisher, April, 186–188
Percentage—noun or adjective? David Rappaport, Jan., 25–26
Planning an arithmetic fair, Dorothy F. Gilman and Members of the Faculty, Dec., 428–430
Those problem-solving perplexities, Cleata B. Thorpe, April, 152–156
Problems without numbers, Walter L. Klas, Jan., 19–20
Professional dates, Jan., 45; Feb., 82; Mar., 140; April, 197; May, 266; Oct., 326; Nov., 362; Dec., 403
Projects make mathematics more interesting, John B. Haggerty, April, 172–175
A Psalm of Lamentation or The Psalm of the Empty Set, Nellie Culp, Oct., 296
Relationship between arithmetic achievement and item performance on the Revised Stanford-Binet Scale, Aileen Shine, Feb., 57–59
Remainders in division and a floor number line, Edwina Deans, assistant editor, Mar., 131–134
Research on elementary-school mathematics—1960, J. Fred Weaver, assistant editor, May, 255–260; Oct., 301–306
Reviews, books and materials, Clarence Ethel Hardgrove, assistant editor, Jan., 36–38; Feb., 83–85; Mar., 138–140; April, 196; May, 261–264; Oct., 307–308; Nov., 377; Dec., 417, 440–441
San Diego's new arithmetic program, J. Fred Wilson, William Self, and Don Chamberlin, Dec., 422–424
The School Mathematics Study Group Project on Elementary-School Mathematics, J. Fred Weaver, assistant editor, Jan., 32–35; Dec., 436–439
Sets and numbers in grade one, 1959–60, Patrick Suppes and Blair A. McKnight, Oct., 287–290
Some basic geometric ideas for the elementary teacher, Leon Rutland and Max Hosier, Nov., 357–362
Some new approaches to old arithmetic, Emilio Rivera, Oct., 291–294
A spark of enthusiasm, Lucile B. Powell, May, 239–241
Subtraction by complement-addition-complement, Romae J. Cormier, Nov., 344
Summer institutes in mathematics and science-mathematics for elementary-school personnel, Feb., 90
Teacher exchange program opportunities, 1962–63, May, 233
Teaching creativity in mathematics, Franz E. Hohn, Mar., 102–106
Teaching multiplication facts need not be grim, Esther R. Unkel, Oct., 297–299
Tease them to think, Thomas J. Jennings, May, 246–247
Testing—without tests, K. L. Harrison, Jan., 21
A third grade enjoys arithmetic, Edwina Deans, assistant editor, May, 251–254
A time for decision, Ben A. Sueltz, Oct., 274–280
Understanding and the ability to solve problems, Angela Pace, May, 226–233
University students' comprehension of arithmetical concepts, Wilbur H. Dutton, Feb., 60–64
Unusual arithmetic, Cynthia Parsons, Feb., 69–74
The use of models in the teaching of mathematics, Roger Osborn, Jan., 22–24
The use of overlay charts, Ruth Machlin, Dec., 433–435
The use of "1" in building concepts, Frank C. Araniti, Oct., 299–300
The versatile number runner, Mary Michalov, April, 182–185
Why do pupils avoid mathematics in high school? Guy M. Wilson, April, 168–171
Will you contribute to a forthcoming yearbook? Julius H. Hlavaty, Jan., 20; Feb., 68

*The one* ***arithmetic*** *program*
*best equipped to provide*
***successful arithmetic teaching***




*1962 Edition*

Grades 3-8, clothbound texts, by Jesse Osborn, Adeline Riefling, and Herbert F. Spitzer. Teacher's Editions for Grades 1 through 8

Grades 1 and 2, text-workbooks by Herbert F. Spitzer and Martha Norman

1. Children EXPERIENCE a true-to-life problem-situation.
2. They EXPLORE ways of solving it by using what they already have learned about arithmetic.
3. Questions and exercises help pupils DISCOVER for themselves the reasons for new arithmetic steps.
4. Pupils DEVELOP deeper understanding.

**WEBSTER PUBLISHING COMPANY**
1154 RECO AVE. ST. LOUIS 26, MISSOURI

*With*

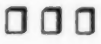
# Seeing Through Arithmetic

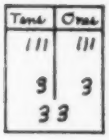
(Grades 3-6)

*teachers have new ways of helping children see through*

concepts computation problem solving

**Here are three examples.**






3 is sometimes a name for 3 ones. 3 is sometimes a name for 3 tens. The meaning of 33 depends on the principle of place value in our base-ten numeration system.

By including the zero in the second partial product, it's easy for pupils to see that 3312 is the sum of 432 and 2880.

72
46 ← Think of 46 as 40 and 6.
432 ← Number in 6 groups of 72.
2880 ← Number in 40 groups of 72.
3312 ← Number in 46 groups of 72.

The speed of sound is about 1100 feet per second. In about how many seconds will sound travel 5500 feet?

$\frac{1100}{1} = \frac{5500}{n}$

A ratio equation describes the problem situation in *mathematical* language. Then:

$\frac{1100}{1} = \frac{5500}{5}$ ← So $n$ must be $5 \times 1$.

5500 is $5 \times 1100$.

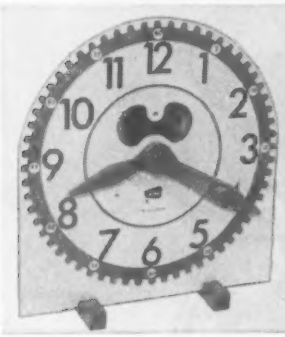
Sound travels 5500 feet in about 5 seconds.

**Free on request:** 32-page booklet of sample lessons. Ask for #490.

**SCOTT, FORESMAN AND COMPANY**

Chicago 11 • Atlanta 5 • Dallas 2 • Palo Alto • Fair Lawn, N.J.

A CLASSROOM CLASSIC by Judy



WHAT MAKES IT WORK?

*Students learn time-telling quickly with the Judy Clock!*

Now standard in many classrooms, the Judy clock teaches time-telling, and shows "what makes it work." Movable plastic hands, connected by visible plastic gears, show minute-to-hour relationship. Colorful markings and numerals on durable 14" hardboard face. Bases detachable for storage. $2.95, F.O.B. Minneapolis.

Available for purchase under the National Defense Education Act, title III.

Write for complete catalog.

**THE JUDY COMPANY**

Dept. AT-1261, 310 N. 2nd St., Minneapolis 1, Minn.

**CREATIVE TEACHING TOOLS FROM STERLING**

**MATH WITHOUT NUMBERS** by Edgar S. Bley. An ingenious new teaching method that clarifies both fundamental and more advanced concepts. It presents geometry and set theory through simple drafting techniques. $2.57 net, reinforced library binding Ages 12 up

**HOW TO USE THE ARITHMETIC YOU KNOW** by Geoffrey Mott-Smith. "A handy guide that reviews basic arithmetic processes, points out short cuts to calculations and suggests efficient application of mathematics in everyday affairs."—ALA Booklist
$2.95 Ages 10 up

**HOW TO USE ALGEBRA IN EVERYDAY LIFE** by the Editors of Sterling. "After a brief refresher in algebra, the book shows how to apply elementary algebra to various fields such as sports, nature, travel."—Publishers' Weekly
$3.95 Ages 12 up

**101 MATHEMATICAL PUZZLES (and How to Solve Them)** by Don Reinfeld and David Rice. "Each puzzle is clearly explained so that you can follow the logical development of its solution. Your mind will be sharpened by the stimulation of logical thinking."—Bulletin, Assn. of Secondary School Principals
$2.50 Ages 12 up

Order from Your Jobber or from Dept. AT

**STERLING PUBLISHING CO., INC.**

419 Fourth Ave., New York 16, N.Y.

**A GUIDE TO THE USE AND PROCUREMENT OF TEACHING AIDS FOR MATHEMATICS**

**by Emil J. Berger and Donovan A. Johnson**

Discusses the function, use, and selection of teaching aids and equipment for mathematics.

Gives a comprehensive listing, with items classified according to desirability.

Gives estimated prices and sources of supply.

Contains bibliographies of library and reference materials.

Arithmetic teachers will be especially interested in:

Construction Materials and Tools
Arithmetic Devices and Models
Toys and Enrichment Material
Games for Arithmetic
Library Lists

*48 pages.* *75¢ each.*

*Postpaid if you send remittance with order.*

**NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS**

**1201 Sixteenth Street, N.W.** **Washington 6, D.C.**

*Here's something to start them talking—*

## LEARNING TO USE ARITHMETIC, REVISED

**for Grades 1-8**

Fehr Gunderson Hollister Phillips
Randall Urbancek Wren Wrightstone

---

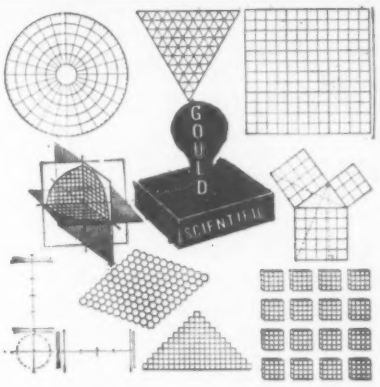
After all, it's in the elementary grades that children learn what mathematics is all about. They're laying the foundation for what they're going to learn later on, not only in mathematics but also in science, technology, and many other fields. That's why it's so important to have an up-to-the-minute arithmetic program—one that's keyed to the latest trends in mathematics teaching.

LEARNING TO USE ARITHMETIC, REVISED is all that and more—it's also fun! Fun for the teacher because the program offers such wonderful teaching aids: workbooks for each grade, teachers' editions of both the textbooks and workbooks, special charts and manipulative materials, and separate achievement tests. Fun for the pupils because there's so much to *do,* and the lessons are all about things they're interested in and care about.

Yes, there are plenty of reasons to get excited about this new revised program. *It's coming in January.*

**D. C. HEATH AND COMPANY**

Home Office: BOSTON 16 Sales Offices: ENGLEWOOD, N.J. CHICAGO 16
SAN FRANCISCO 5 ATLANTA 3 DALLAS 1 LONDON W.C. 1 TORONTO 2-B




***New GRAPH STAMPS and Ditto, Acetate Ink STAMP Pads***

Now you can cut your paper handling in half and save time and labor in grading tests and worksheets. Let these new Ditto stamp pads work for you. Simply stamp directly on the master sheet and run off 200 copies. Stamps & Pads (Purple, Green, Red) $3.00 each. Postpaid. (Acetate to print on vinyl for overhead projectors.)

Over 70 new (Big 3" size) graph stamp patterns to choose from. Select your school year needs now. Patterns include games, 3 & 4 dimension TicTacToe, Hex, Tangrams, etc., as well as clock, bar, pie, lattice, linear, log, polar, triangle, hyperbolic, probability, statistical, vector, calculus, geometry, algebra, trig, map making, geological, astronomy. Circular slide rule and nomograph (kit) graph stamps. Send for your Catalog "AMS" Today.

**GOULD SCIENTIFIC** P.O. Box 6743 Wash. 20, D.C.

THE ELEMENTARY AND JUNIOR HIGH SCHOOL MATHEMATICS LIBRARY

*by* CLARENCE ETHEL HARDGROVE

An annotated bibliography of selected recreational and informational materials for mathematics.
Classified by primary grades, intermediate grades, and junior high school.
Suggests readings to help your students grow in the ability to think with the ideas of mathematics.
32 pages 35¢ each

*Postpaid if you send remittance with order.*

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
1201 Sixteenth Street, N.W. Washington 6, D.C.

THE NEW
WINSTON
ARITHMETICS
SERIES

*Brueckner, Merton, Grossnickle*

*Now includes both* **Standard** *and* **Extended Programs** *for Grades 1-9 (Extended Program, Grades 7-9, ready early in 1962).*

*The* **Winston Arithmetics** *make abundant use of manipulative and visual experiences to develop concepts as they are introduced. Problems are built around situations with which most children would be familiar. In the* **Extended Program,** *the emphasis is on mathematical principles, with extension units designed to lead able students toward higher levels of abstract appreciation.*

The John C. Winston Co., *a division of*

**HOLT, RINEHART AND WINSTON, INC.**
383 Madison Avenue New York 17, N.Y.

# *the Ouija Board, you say . . .*

is as good a way as any to pick an arithmetic program because they're practically all alike. Alike in some respects, yes, but widely different in others—and the differences are what distinguish the Ginn Arithmetic Program.

The Ginn Arithmetic Program for grades 1-8, and its accompanying materials, comprise a complete, hard-hitting program that has been instrumental in the development of thousands of successful mathematics students.

This is a program that instructs not just your average pupils but the slow and superior pupils as well. Here you'll find the exercises that provide extra practice, that reinforce understanding, and that put across the nature and function of mathematical processes to slower learners. Here, too, is the work that challenges and builds independent thinkers.

For grades 1 and 2, *Numbers We Need* (write-in texts and Teachers' Editions) by Brownell and Weaver . . . a real program in number thinking, not just picture books.

For grades 3 to 8, *Arithmetic We Need, Enlarged* (textbooks, workbooks, tests and manuals) by Buswell, Brownell, Sauble . . . comprehensive.

For grades 4, 5, and 6, *The Ginn Arithmetic Enrichment Program* by Marks, Smart, Brownell, and Sauble. These three new write-in texts, *Exploring Mathematical Ideas, Enlarging Mathematical Ideas,* and *Extending Mathematical Ideas* provide much-needed extension work in basic mathematics plus insights into modern mathematics.

For grades 7 and 8, *Number Principles and Patterns* (68-page pamphlet) by Archer, and *Geometry, Discovery by Drawing and Measurement* by Bentley and Potts.

Teaching aids include the ingenious *Ginn Arithme-Sticks* (a multiple-use abacus and counting device) and Ginn Number Cards.

*ginn and company*

Home Office: Boston
Sales Offices: New York 11
Chicago 6 Atlanta 5
Dallas 1 Palo Alto
Toronto 16

## MATHEMATICS TESTS AVAILABLE IN THE UNITED STATES

*by*

SHELDON S. MYERS

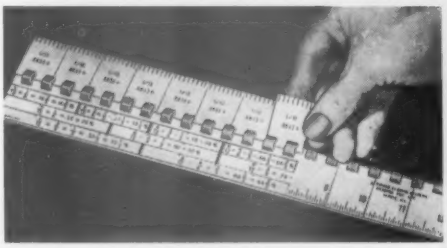
A listing, as complete as possible, of all the mathematics tests available in the United States. Gives information as follows: name of test, author, grade levels and forms, availability of norms, publisher, and reference in which review of test can be found.

*16 pages* *50¢ each*

*Postpaid if you send remittance with order.*

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
1201 Sixteenth Street, N.W.
Washington 6, D.C.

## HINGED TABS ON RULER TEACH CHILD FRACTIONS



**One-foot ruler and hinged parts printed with many fractional equivalents, including decimals and per cents. These can be manipulated into many combinations. By proper handling it is possible to improve each child's understanding of fractions and how to use them in the four fundamental math processes. Demonstrator ruler also available can stand upright on desk or table, or hang on wall. Instruction manual accompanies each ruler.**

***INDIVIDUAL STUDENT RULERS . . . $1.50 EA.***
***TEACHER'S CLASSROOM DEMONSTRATION RULER (4' x 8") . . . . . $15.00 EA.***

**HELBERG ENTERPRISES, INC.**
***8327 N. Lawndale Ave.***
***Skokie, Ill.***

# EVALUATION IN MATHEMATICS

**26th NCTM Yearbook**

Written to help mathematics teachers improve their techniques of evaluating achievement.

Applies basic principles of measurement to the evaluation of achievement in mathematics.

Offers a wealth of ideas that may be applied in the reader's particular situation.

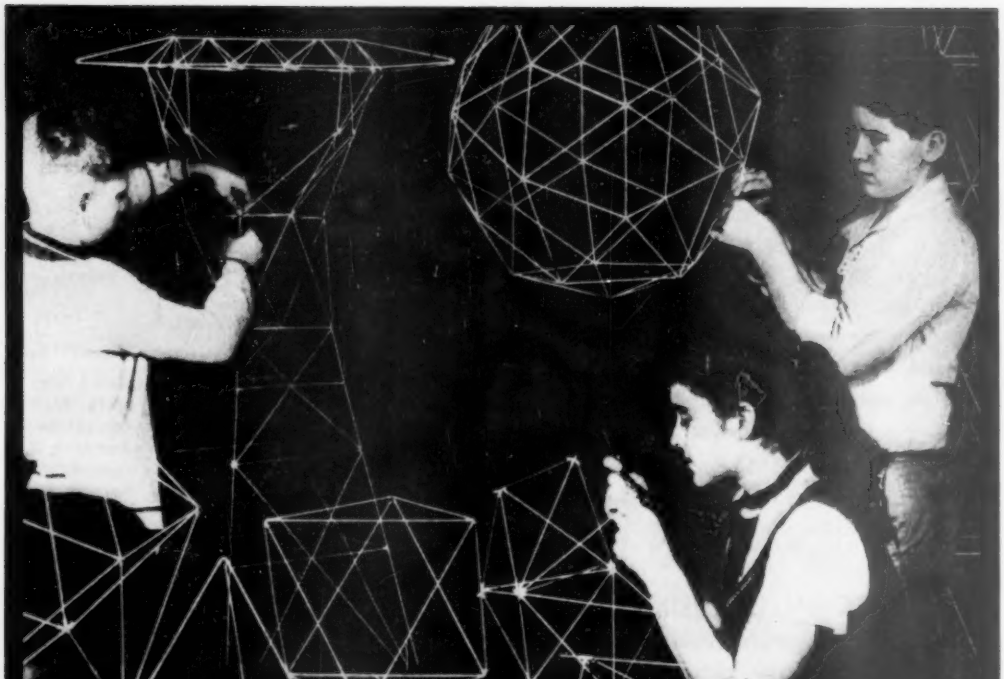
Developed around three main themes: general theory, practical techniques, and application to classroom problems.

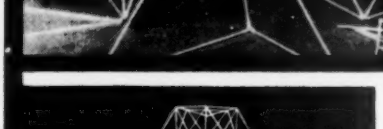
Topics include: role of evaluation in the classroom, basic principles, constructing tests, interpreting scores, published materials, evaluation practices, appraising attitudes, interpreting records, etc.

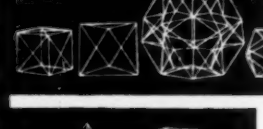
**220 pp. $3.00 ($2.00 to members of the Council)**

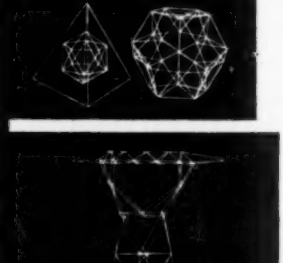
***Postpaid if you send remittance with order.***

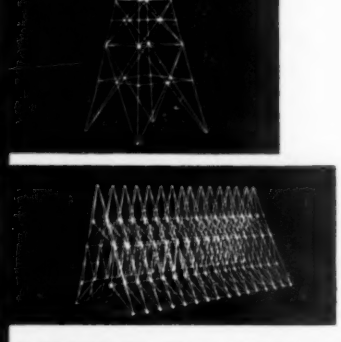
**NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS**
**1201 Sixteenth Street, N.W.** **Washington 6, D.C.**











*at school — at home*

# BUILD & LEARN with D-STIX

**POPULAR VISUAL TOOL IN HIGH SCHOOLS AND COLLEGES**

Construction kits for assembling geometric structures of the present and future. Widely used as visual tools for high school and college instructors, architects and structural engineers. Utilizes universal-type plastic connector and non-toxic color dyed "D-Stix."

**PRE-ENGINEERING KIT NO. 701**

For students and adults. Those with advanced interest in geometric structures and artistic design.

**APPRENTICE KIT NO. 501**

Ages 9 to adult. All major structures in solid geometry can be assembled. Perfect for hobbyists.

**JUNIOR KIT NO. 301**

Ages 6 to 16. Kit contains plans for building three of the five basic polyhedrons . . . numerous toy versions of launching towers, rockets.

Can be purchased under Title 3, National Defense Education Act.

**ORDER BY MAIL**

Northwest Vocational Sales
P.O. Box D-1, Yardley, Wash.

Gentlemen: Please ship me the following order of D-Stix:

| Quantity | | Amount |
|---|---|---|
| .....Kit No. 701 | @ $7 ea. | ..... |
| .....Kit No. 501 | @ $5 ea. | ..... |
| .....Kit No. 301 | @ $3 ea. | ..... |
| | Total | ..... |

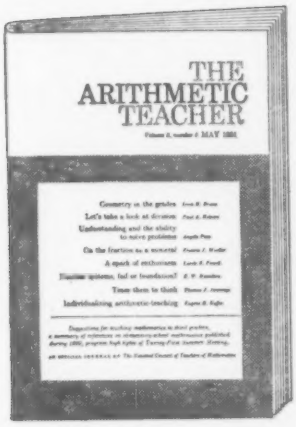
Washington residents add 4% sales tax.

Name.....

Address.....

City..... State.....

Remittance encl. ☐ Ship C.O.D. ☐




# An official journal of the National Council of Teachers of Mathematics

**THE ARITHMETIC TEACHER,** founded only eight years ago, is now received by more than 18,000 subscribers.

- Brings you the best in arithmetic instruction.
- Devoted to the improvement of the teaching of arithmetic in kindergarten and the elementary school.
- Presents information regularly on teaching and curriculum problems, classroom ideas, investigations and research, testing and evaluation, teaching aids and devices, and reviews of new publications and materials.
- Features articles by leading thinkers in this field.
- Published monthly, October through May.

Elementary school principals are invited to order institutional subscriptions for their teaching staffs.

Membership-subscriptions (for individuals only) include membership in the National Council of Teachers of Mathematics.

*If you are already a subscriber, tear out this sheet and give it to a friend.*

*If you are already a subscriber, please pass this invitation on to a colleague.*

**Order for the ARITHMETIC TEACHER**

National Council of Teachers of Mathematics
1201 Sixteenth Street, N.W., Washington 6, D.C.

Please enter an order for:

☐ Institutional subscription ..... $7.00
(For schools, libraries, departments, and other institutions)

☐ Membership-subscription .....$5.00

*Add 50¢ for foreign mailing, 25¢ for Canadian mailing.*

I enclose check, cash, or money order for $..... Please bill me .....

Please send me a sample copy .....

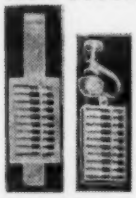
Name (Please print) .....

Street Address .....

City ..... Zone ..... State .....

# EDMUND INSTRUCTIVES

## Now . . . FASCINATING ABACUS JEWELRY



A gem of an idea! Imported sterling silver earrings and tie clip, with movable counters. A fascinating "trade mark" for any math teacher. Also the perfect gift for math students and other math-minded persons. Only ⅞" long.

**Stock No. 40,422-SJ Earrings ..$2.97 Postpaid (tax incl.)**

**Stock No. 40,423-SJ Tie Clip ..$1.95 Postpaid (tax incl.)**

## 3-D DESIGN WITH SPACE SPIDER

Create 3-dimensional colored designs that "float in space" simply by weaving fluorescent stands between different planes of "shadow box." Design abstractions, plane and solid geometrical figures. For home decoration or educational use. Kit contains 3 black notched wood panels (the background), each 6½" sq., 3 reels fluorescent thread, black clips, needle, instruction booklet.

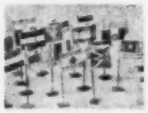
**Stock No. 70,278-SJ .....$2.95 Postpaid**

## SPECIAL FOR SCHOOLS AND GROUPS

Save ½—order a set of six Space Spiders.
**Stock No. 70,297-SJ .....$13.95 Postpaid**

### NEW! CIRCULAR SLIDE RULE: Pocket Size—Fast—Easy to Use!

Be a Math Whiz! New Circular Slide Rule multiplies, divides, figures fractions, percentages, squares, cubes, roots, proportions, circumferences, areas, retail prices, fuel consumption. Eliminates the confusions of ordinary slide-rules. Priced far lower. Faster, easier to learn and use. Constructed of a aluminum discs with plastic indicator. 3½" diameter. Directions included.
**Stock No. 30,338-SJ .....$4.95 Postpaid**



### 100 UNITED NATIONS FLAGS —TEACHING AID AT 5¢ EACH

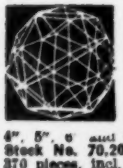
Would you recognize the flags of Mali, Yemen, Togo, Gabon, Chad, Dahomey and Byelorussian among those of 100 flags belonging to members of the United Nations? Ideal gift for anyone interested in world affairs. 2" x 2¾", printed in color on heavy silk-like fabric. Each is identified and mounted on wood staff with removable base. Pliofilm envelope protects each flag.
**Stock No. 70,503-SJ .....$5.00 Postpaid**

### ABACUS

Our abacus is just the thing to use in class to teach place value and number concepts. It is more effective than a place frame. It is of our own design and is 8¾" wide and 7⅞" long. It is made of a beautiful walnut wood, with 6 rows of 10 counters. Complete instructions are included with every abacus.
**Stock No. 70,201-SJ .....$4.95 Postpaid**

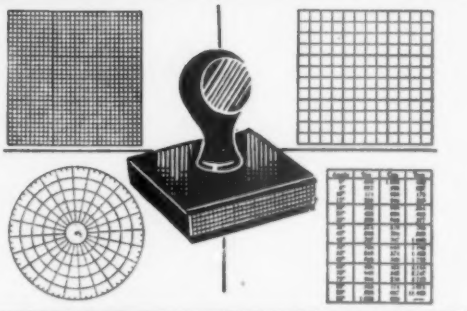
## TEACHING AIDS

### D-STIX CONSTRUCTION KITS



D-Stix clarify geometric figures for the young by actually demonstrating them in three dimensions. They make recognition and understanding fun. They can also be used for showing molecular structures in science classes, concepts of elementary science and physics. These modern construction kits are far superior to older style wood or metal construction kits.

230 pieces, 5, 6 and 8 sleeve connectors, 2", 3", 4", 5", 6 and 8" colored D-Stix—
**Stock No. 70,209-SJ .....$3.00 Postpaid**
370 pieces, incl. 5, 6, and 8 sleeve connectors. 2", 3", 4", 5", 6", 8", 10" and 12" D-Stix in colors—
**Stock No. 70,210-SJ .....$5.00 Postpaid**
452 pieces, includes all items in 70,210 above plus long unpainted D-Stix for use in making your own designs—
**Stock No. 70,211-SJ .....$7.00 Postpaid**



## GRAPH RUBBER STAMPS

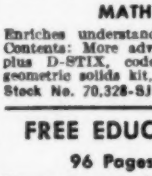
Teachers whose tests require a graph background will find these 3" square rubber stamps real time and labor savers. Grading becomes **easier when students** have these stamped graph patterns to do their test examples on. There are no extra sheets to become separated and lost, either.

**100-Block-per-Square-Inch Graph Stamp**
Stock No. 50,255-SJ—size 3" .....$3.00 Postpaid
**¼"-Inch Grid Graph Stamp**
Stock No. 50,351-SC—size 3" .....$3.00 Postpaid
**Polar Coordinate Graph Stamp**
Stock No. 50,359-SJ—size 3" .....$3.00 Postpaid
**Trig Functions Graph Stamp**
Stock No. 60,115-SJ—size 3" .....$3.00 Postpaid
**Felt Stamp Pad**
Stock No. 60,134-SJ—6½" x 4¼" inked .....$1.50 Postpaid

## ANALOG COMPUTER KIT

Ideal introduction to the increasingly important electronic computer field. For bright students, or anyone interested in this new science. Demonstrates basic analog computing principles—can be used for multiplication, division, powers, roots, log operations, trig problems, physics formulae, electricity and magnetism problems. Easily assembled with screwdriver and pliers. Operates on 2 flashlight batteries. Electric meter and 3 potentiometers are mounted on die-cut box. Answer is indicated on dial. Computer is 20" long, 9" wide, 2" deep.
**Stock No. 70,341-SJ .....$14.95 Postpaid**

## MATH LEARNING AIDS ADD NEW VISTAS IN TEACHING TECHNIQUES



Stimulation, challenge, enrichment is the substance of these two Math Enrichment Chests for students, teachers. Makes teaching and learning math fun. Fits in with "math lab" concept. Endlessly versatile. Helps slow learners, enriches course for gifted children.

### MATH CHEST "A" (primary)

Helps beginners grasp first principles. Contents: Design compass, magnetic compass, adding machine, counting jump rope, desk clock, code maker, desk abacus, 100 colored 1" cubes, devices for making geometric forms, a "think" puzzle.
**Stock No. 70,327-SJ .....$10.00 Postpaid**

### MATH CHEST "B" (intermediary)

Enriches understanding, adds "spice" on higher math plane. Contents: More advanced versions of some items in Chest "A" plus D-STIX, code-maker, drawing and magnetic compasses, geometric solids kit, math enrichment sheets.
**Stock No. 70,328-SJ .....$10.00 Postpaid**

## FREE EDUCATIONAL CATALOG—"SJ"

### 96 Pages . . . New, Useful Instructives

Dozens of new devices for teaching mathematics, science, astronomy, physics, etc. Scores of new kits and materials for Science Fair and other science projects! That's what you'll find in Edmund's new 96-page Educational Catalog for 1961. It's literally packed with new and exclusive math-and-science concepts developed by Edmund Scientific Co. and now available to schools for the first time.

Edmund's well known values in science equipment—telescopes, microscopes, optical materials, and many other school requirements—are also included in this Catalog. For standard values and exciting new offerings.

write for **Edmund's 1961 Educational Catalog—"SJ" today**

ORDER BY STOCK NUMBER . . . SEND CHECK OR MONEY ORDER . . . SATISFACTION GUARANTEED!

**EDMUND SCIENTIFIC CO.** BARRINGTON, NEW JERSEY